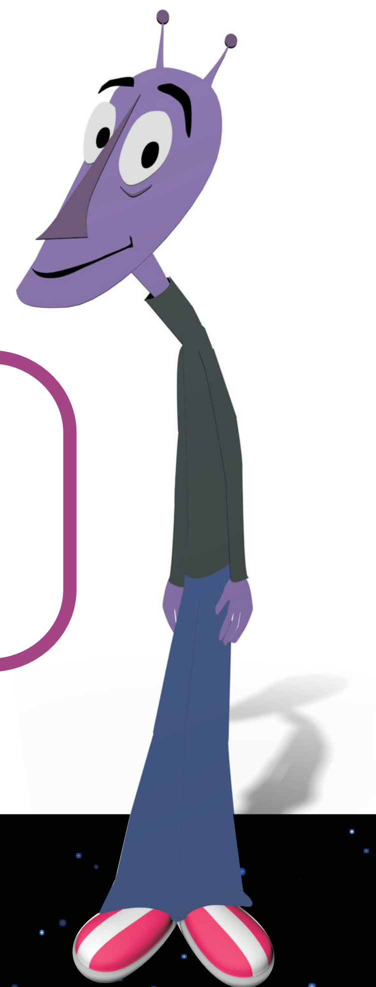
Geometry

Module 11

Transformation of Shapes

Lesson 1
Translations and Reflections



Teacher Notes 11.1

Objectives

- Perform translations and reflections of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).
- Draw and describe the results of translations and reflections about the *x*- and *y*-axis.

Prerequisites

Plotting points on the coordinate plane

Adding and subtracting integers

Vocabulary

Transformation
Translation
Reflection
Line of reflection
Image
x-coordinate (10.1)
y-coordinate (10.1)
Motion rule
Ordered pair (10.1)
Mirror image
x-axis (10.1)
y-axis (10.1)

Get Started

- Sketch the first quadrant of the coordinate plane on the board. Extend the axes 10 units in each direction. Teachers should choose a point in the quadrant without telling the students the coordinates of the point.

- Tell students they are to guess the coordinates of the point in the first quadrant. Ask for a volunteer to guess the coordinates. If they are correct, congratulate them and choose another point for someone else to guess its location. If they are incorrect, tell them so and give them a rule for finding the point from the coordinates that were guessed. For example, *to get to my point from there, move three units left and two units down*.

- Have the students use this rule to find the location. Repeat with a hidden point using all four quadrants. Then, have a student choose a point while the teacher guesses its location. If wrong, have the student make a rule for him or her to find it.

Subtopic 1 Translations

Expand Their Horizons

In this subtopic, students learn the first of three basic rigid motions of geometry, translations. A basic rigid motion is a transformation or change that does not change the size or shape of the figure. A translation is a *slide*. Take a piece of paper and slide it around the chalk board. The paper does not change size or shape or even orientation. The top left corner is always the top left corner and so on.

A figure can be slid in any direction. On a coordinate plane, adding to the x-coordinate moves the figure right; subtracting from the x-coordinate moves the figure left. When translating up or down, the y-coordinate changes. Adding to the y-coordinate moves the figure up; subtracting from the y-coordinate moves the figure down. These rules should be intuitive to students because they parallel the rules for plotting and locating points.

Motion rules are shorthand ways to describe a transformation. Instead of writing *move the point three units right and six units down*, the directions can be written as $(x, y) \rightarrow (x + 3, y - 6)$. The arrow translates into the phrase *goes to*.

After a point or figure is translated, it is referred to as the *image*. The original point or figure can be referred to as the original, or the *pre-image*. Image points are labeled by adding the prime symbol after the letter that labels it. For instance, if A is the pre-image point, A' is the image point.

1 The size, shape, and orientation did not change. The figure was slid down and to the right. It is a translation.

2 While the size and shape are the same, the orientation has changed. For instance, Point A moved from the top of the figure to the bottom of the figure. It is not a translation.

3 Subtract four from each x-coordinate and three from each y-coordinate.

4 Compare the coordinates of A and A'. To get from A to A', move two units left and three units up. Notice that this works to get from *any* point in the original figure to its corresponding point in the image figure. The motion rule is $(x, y) \rightarrow (x - 2, y + 3)$.

Additional Examples

1. **The point located at (5, 1) is translated eight units down. What are the coordinates of the image point?**

   A slide up or down is a change in the y-coordinate. Since the movement is down, subtract:
   $(5, 1) \rightarrow (5, 1-8) \rightarrow (5, -7)$.

2. **Write the motion rule for the translation that moved $H(2, -7)$ to $H'(3, 5)$.**

   The x-coordinate increased by one: $x + 1$. The y-coordinate increased by 12: $y + 12$.

   The rule is $(x, y) \rightarrow (x + 1, y + 12)$.

Subtopic 2 Reflections

Expand Their Horizons

In this subtopic, students learn that a reflection is another transformation in which the pre-image and image remain congruent. Unlike with translations, however, the figure will change in orientation. Mark the top left corner of a piece of paper (on both sides) and draw a horizontal line on the chalkboard. Flip the paper over the line. The top left corner becomes the bottom left corner and so on.

When a figure is reflected over the x-axis, the x-coordinate remains the same, and the y-coordinate becomes its opposite. When a figure is reflected over the y-axis, the y-coordinate remains the same, and the x-coordinate becomes its opposite.

Common Error Alert:

Students often confuse the motion rules for reflections. Because the x-axis is horizontal, they imagine a figure moving horizontally when a figure is reflected over the x-axis. Likewise, when they are directed to reflect a figure over the y-axis, they may automatically think "up and down" and picture the image figure above or below the original figure. It might help students to sketch and label a set of axes at the top of their page and to look at that when determining image points for a reflection.

Motion rules can be written for reflections. To reflect over the x-axis, for instance, the rule is $(x, y) \rightarrow (x, -y)$. Remind students that the negative sign means "opposite;" so if the y-coordinate in the pre-image point is negative, it becomes positive in the image. In other words, either coordinate can be either positive or negative.

After one transformation is performed, one prime symbol is used. If the image of one transformation is then used as the pre-image of another transformation, the image points are labeled with two prime symbols. The label A'' is read as *A double prime*.

5 The transformation is not a reflection because the orientation has not changed. The transformation shown is a translation.

6 The transformation is a reflection across a vertical line. Each pre-image and image point is equidistant from the line of reflection.

7 To reflect across the *y*-axis, take the opposite of each *x*-coordinate. If the coordinate grid were folded along the *y*-axis, the two figures would match up exactly.

8 Look at M and M'. The difference in the ordered pairs is that the *y*-coordinates are opposites. This is true at every vertex. The motion rule is $(x, y) \rightarrow (x, -y)$.

Additional Examples

1. **The point (5, 6) is reflected across the *x*-axis. What are the coordinates of the image point?**

   To reflect a point across the *x*-axis, just take the opposite of the *y*-coordinate: (5, -6).

2. **A point was reflected across an axis, and the coordinates of the image were (-1, 8). If the coordinates of the original point were (1, 8), which axis served as the line of reflection?**

   Because the *x*-coordinate became its opposite and the *y*-coordinate did not change at all, the line of reflection was the *y*-axis.

Look Beyond

Translations and reflections are two of the four transformations students will study in this program. They are both isometries, which means that the pre-image and image are congruent. They will study the last isometry, rotations, in the next lesson. After that, they will study dilations, a transformation which changes the size, but not the shape, of the figure.

Connections

Artists use reflections when they draw or paint pictures that include calm water, like a pond or lake. The water line along the shore acts as the line of reflection. To make the picture look natural, the artist must be able to create the mirror image of everything above the water line, which often includes mountains and trees. A good artist can accurately achieve this even when several colors have been used.