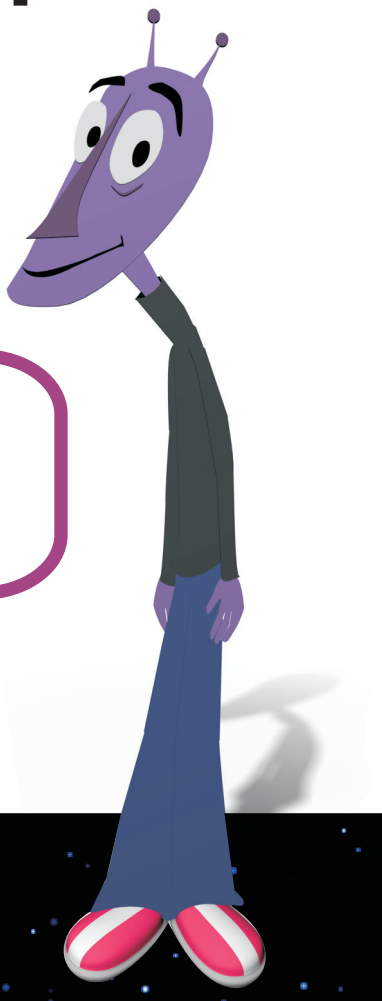
Geometry

★ Module 10 ★

Coordinate Geometry and Spatial Visualization

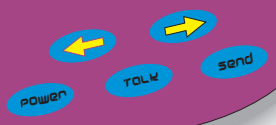


Lesson 5
Building Models

Teacher Notes 10.5

Objectives

- Identify two-dimensional patterns (nets) for three-dimensional solids, such as prisms, pyramids, cylinders, and cones.
- Build three-dimensional solids from two-dimensional patterns (nets).
- Recognize the front, side, and top views of three-dimensional figures.
- Sketch front, top, and side views of three-dimensional figures with or without technology.



Prerequisite

Identifying and classifying three-dimensional figures including prisms and pyramids

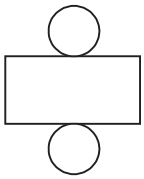
Vocabulary

- Net
- Three-dimensional (10.4)
- Two-dimensional (9.1)
- Prism (10.4)
- Cube (10.4)
- Lateral face (10.4)
- Pyramid (10.4)
- Polyhedron (10.4)
- Convex polyhedron (10.4)
- Cylinder (10.4)
- Cone (10.4)
- Tetrahedron (10.4)

Get Started

- Before class, prepare paper cutouts of one rectangle and two congruent circles so that the circumference of the circle is equal to the length of the rectangle. (It will be easiest to cut the circles first, measure or calculate the circumference, and then cut out the rectangle.)
- At the start of class, hold up the figures and ask students if they know what three-dimensional shape can be made from these three two-dimensional figures. Show that a cylinder can be formed by bending the rectangular paper so that the two shorter sides meet. Tape them in place. Then, tape the two circles on each end to form the bases.

- Untape and unfold the figure just enough so that it looks like the figure shown below. Tell students that this figure is a net and that they will learn more about nets in the lesson.

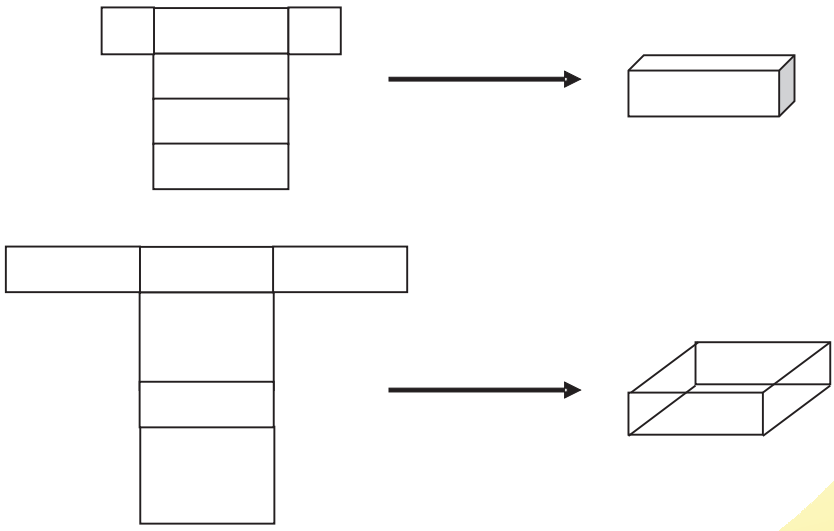


Subtopic 1 Nets for Three-Dimensional Figures

Expand Their Horizons

In this subtopic, students identify solids from their nets and draw nets for solids. A net is a two-dimensional pattern of a three-dimensional object. When it is folded, it forms the object. There can be more than one net for the same solid. It may help students if teachers bring in some boxes (cereal boxes, shoeboxes, mailing boxes) for them to cut and unfold into nets. Paper food containers in shapes of other polyhedra are more difficult to find but make great additions to the learning environment, if available.

Encourage students to draw their solids in proper proportion to their given nets. For example, both nets below are nets of rectangular prisms, but the prisms have clearly different shapes.



Sometimes in a net, dotted lines are used to show hidden edges in the folded figure. This is optional. The use of dashed lines in nets and/or solids should be based on students' ability levels.

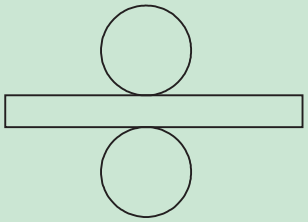
1. Visualize cutting and unfolding the object. Make sure all six faces of the rectangular prism are represented in the net.

2. Fold the circle up and the other face around so the left and right sides meet. The figure is a cone.

3. Fold up the four triangular faces so that the top vertices intersect. This point is the vertex of a square pyramid.

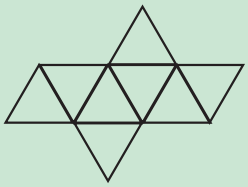
Additional Examples

1. Draw the solid figure this net represents.



The net forms a wide and short cylinder.

2. What solid figure does this net represent?



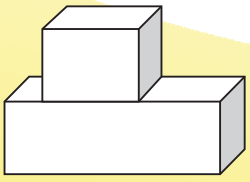
The eight congruent triangles fold into an octahedron.

Subtopic 2 — Views of Three-Dimensional Figures

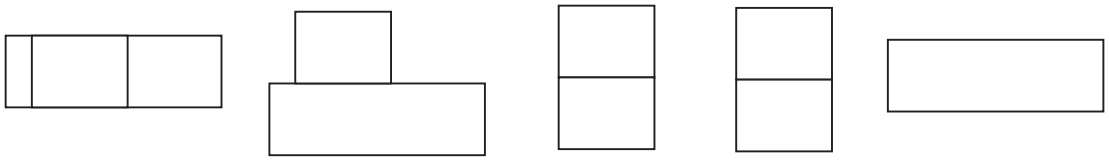
Expand Their Horizons

In this subtopic, students draw the top, front, and side-views of three-dimensional figures. They also learn to identify a three-dimensional figure from a set of views. A solid has a total of six views (top, bottom, left, right, front, and back). Depending on the solid, all six views may not be needed to correctly identify it.

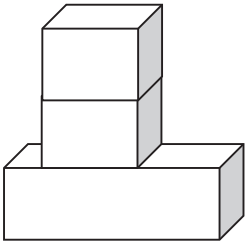
Place solids on a table and have students look at them from each side. Solids may be combined to make more complex figures. For instance, boxes may be stacked on top of each other as shown below.



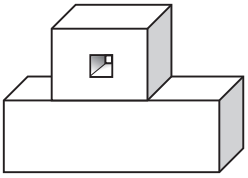
Top: Front: Right: Left: Bottom:



When viewed from the top, the outline of the bottom box can be seen. The two vertical lines indicate that the edges of the smaller box can also be seen. This shows the location of the top box. If this were the only view given, students would not necessarily draw the correct solid because the same top view would be seen for the following solid as well.

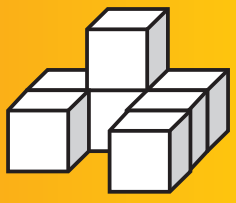


The side views indicate that there is only one small box on top of the larger box. The front view is also needed because the solid with the cube-shaped hole below would have the same top and side views as our original solid.

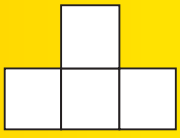


Common Error Alert:

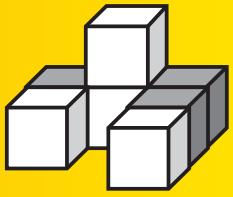
Students often struggle with the idea that the depth of a figure is not given from any single view. Consider the stack of blocks below. To determine the front view, it may help students to imagine every block being pushed from behind and up against a glass mirror. Blocks that were in the back would be pushed to the front. Three blocks on the bottom row and one block on the second row would be touching the mirror. They form the front view.



Front View:

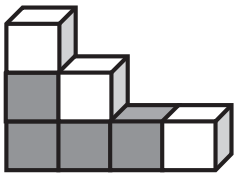


Show students which of the blocks would not be seen by shading them.

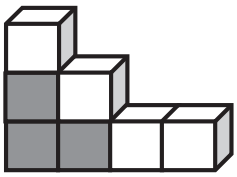


In the lesson, students find the side view of a group of cabinets. The shaded blocks below show which of the cabinets are hidden in the right-side view and the top view.

Right View



Top View



For this lesson, have plenty of graph paper on hand for students to use to draw their solids and nets. It will be especially useful in Problem 2 of Challenge Set 1.

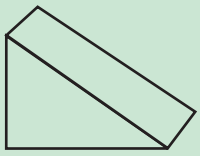
4 Each view is a rectangle. The solid is a rectangular prism whose height is greater than its length and width.

5 The front and right views are rectangles, indicating the solid is a prism. Notice that the "edge lines" coincide with the placement of the vertices in the top view. Since the top view is a hexagon, the solid is a hexagonal prism.

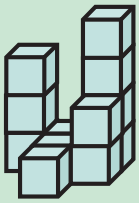
6 From the top, a row of five squares can be seen along with one more square in front of the second one. To form the front view, ignore depth and count how many blocks high each column is. From left to right, the heights are: one, two, three, one, and one. To form the left-side view, imagine moving to the left of the figure and looking directly at it. Students would see a column of blocks three high on the left and a column one block high to the right of it.

Additional Examples

1. **Draw the front, top, and right-side views.**



2. **Draw all six views of the figure.**



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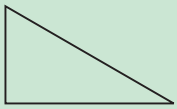
Only the front view indicates that the solid is triangular.

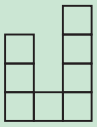
Front:



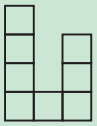
Top:

Right:

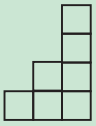
Front:



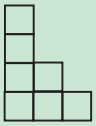
Back:



Right:



Left:



Top:

Bottom:

Look Beyond

In advanced geometry and trigonometry classes, students will learn about cross sections of three-dimensional figures. A cross section is a “slice” of a solid. Parabolas, ellipses, and hyperbolas are all formed from the cross section of a cone or double cone. In art classes, students can study how to draw solids from different perspectives. Most art students learn to draw figures in both one- and two-point perspective.

Connections

Draftsmen must be able to draw a figure in all six of its views to create engineers’ sketches and industrial layouts. Builders of the objects must be able to correctly interpret these drawings. In mechanical drafting, objects are often very detailed with multiple contours, gaps, and curves. It takes much practice and training to be able to correctly conceptualize the multiple two-dimensional views of these objects as one single three-dimensional object, though computer software has made some of this work easier.