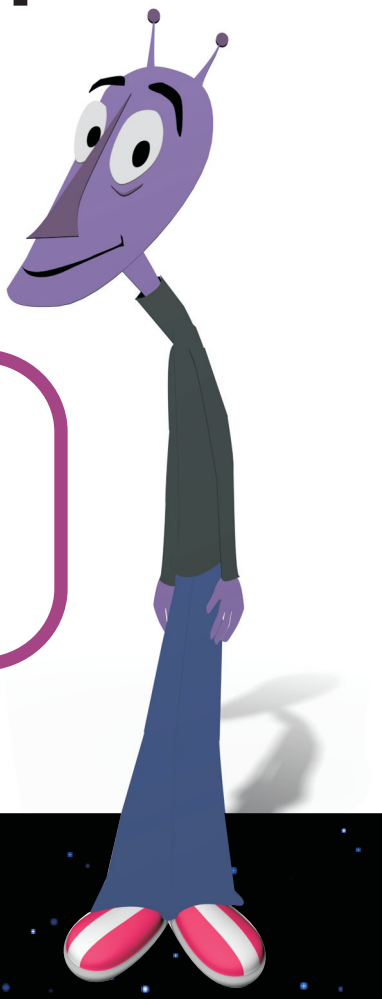
Geometry

★ Module 10 ★

Coordinate Geometry and Spatial Visualization



Lesson 4
Three-Dimensional Shapes

Teacher Notes 10.4

Objectives

- Identify three-dimensional geometric figures using models (rectangular prisms, cylinders, cones, pyramids, and spheres).
- Use properties of standard three-dimensional shapes to identify, to classify, and to describe them.

Prerequisite

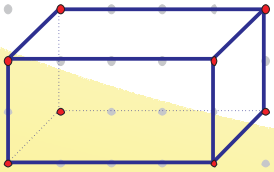
Identifying and classifying a polygon by its number of sides, as convex or concave, and as regular or not regular

Vocabulary

- Polygon (9.1)
- Solid
- Polyhedron
- Three-dimensional
- Cube
- Edge
- Vertex of a polyhedron
- Face
- Platonic solid
- Tetrahedron
- Hexahedron
- Octahedron
- Nonconvex polyhedron
- Dodecahedron
- Icosahedron
- Convex polyhedron
- Regular polyhedron
- Prism
- Base
- Lateral face
- Altitude
- Pyramid
- Sphere
- Equidistant (9.3)
- Cylinder
- Cone

Get Started

Show students how to make a "box" on dot paper. If dot paper is not available, use the intersection of whole number coordinates on a coordinate grid. Then, pass out copies for students to use. Once students have finished their box, give them a few minutes to make their own three-dimensional designs.



Expand Their Horizons

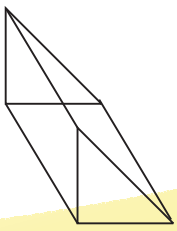
In this subtopic, students learn that a polyhedron is a solid formed from polygonal surfaces, called faces. The sides of the polygons make up the edges of the polyhedron, and the points, where the edges intersect, are called vertices. One polyhedron familiar to students is a "box." Have students identify and count the faces, edges, and vertices of the box and any other polyhedra they drew in the Get Started activity. Teachers can also discuss why any of their other dot-paper drawings are or are not polyhedra.

Similar to polygons, polyhedra can be either convex or nonconvex. In the lesson on polygons, students learned the "rubber band test" for determining whether a polygon was or was not convex. If the rubber band touched each vertex, it was convex. Since there is more than one way to wrap a rubber band around a polyhedron, students can imagine wrapping a cloth around the figure and seeing if the cloth touches every vertex.

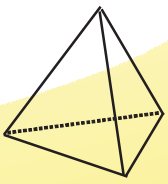
There are five polyhedra that are classified as Platonic solids. A Platonic solid is made up of regular polygons; every face being the same polygon, either a square, triangle, or pentagon. Remind students that regular polygons are both equilateral and equiangular. The Greeks believed that they were the building blocks of nature and assigned an element to each. The tetrahedron, made from four equilateral triangles, was linked to fire. Earth was linked to the cube, or hexahedron, a figure made from six squares. Air was linked to the octahedron, a figure made from eight equilateral triangles. The fourth element, water, was linked to the icosahedron, a figure made from 20 equilateral triangles. The dodecahedron was linked to the planets and stars. It was made from 12 regular pentagons.

Students can build paper models of each of the polyhedra. Many websites offer detailed instructions that can be downloaded.

Two classifications of polyhedra are prisms and pyramids. Prisms have two parallel congruent faces. These faces are joined together by parallelograms, called lateral faces. A pyramid has one base. The lateral faces of a pyramid are triangles. Both prisms and pyramids are named by the shape of their base or bases. For example, a prism with triangular bases is a triangular prism, while a pyramid with a triangular base is a triangular pyramid.



Triangular prism



Triangular pyramid

Common Error Alert:

Students might consider the "bottom" shape to always be the base of a prism. Tell students that figures can be rotated and can be moved about so that is not a valid way to determine the shape of a base. Remind them of the definition of a prism. There are exactly two bases, and they are congruent and parallel. In the prism on the previous page, there are exactly two congruent and parallel triangles, so they are the bases.

The altitude of a prism is a segment, or length of a segment, that extends from one base to the other so that it is perpendicular to both bases. Point out that every lateral edge in a prism is also an altitude. The altitude of a pyramid extends from the base to the vertex of the pyramid so that it is perpendicular to the base. The length of an altitude is the height of the figure.

1 The faces are the surfaces. For a cube there are six: top and bottom, left and right, front and back. The edges are the sides of the polygons. For a cube, it is easiest to count the four on the top, the four on the bottom, and then the four vertical edges on the sides, making a total of 12. The vertices are the points of intersection: four on top and four on the bottom, for a total of eight.

The pyramid has four faces: one triangular base and three triangular lateral faces. There are six edges: three base edges and three lateral edges. There are four vertices: three on the base plus the vertex of the pyramid, where the lateral edges intersect.

2 The first figure has two congruent parallel bases, which are triangles. It is a triangular prism. The second figure has one base, a rectangle, and four lateral faces which are triangles. It is a rectangular pyramid. If the rectangle happens to be a square, it can also be called a square pyramid. The last figure is made up of regular polygons. It is an octahedron.

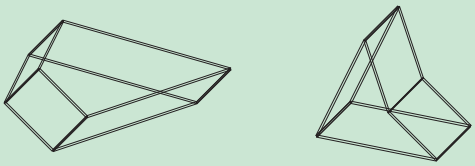
3 A rectangular prism has two parallel congruent rectangles as bases. Draw these first. Then, connect the vertices to form the lateral edges.

4 The first figure is not a polyhedron because its faces are not polygons. The second figure is a polyhedron; it is formed from one rectangle and four triangles.

5 Any two vertices of the figure on the left can be joined to form a line segment. The points that are not endpoints of this segment would not lie on or in the figure, so it is nonconvex. The second figure is convex. A cloth wrapped tightly around it would touch every vertex.

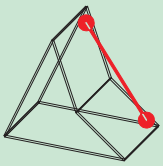
Additional Examples

1. Tell if each figure is nonconvex or convex.



The first figure is convex. Any line segment drawn on or in the figure has all of its points on or in it.

The second figure is nonconvex. A line segment can be drawn such that some of its points are on the figure and some are not.



2. Classify the polyhedra.

The first polyhedron has two congruent parallel bases which have five sides each. It is a pentagonal prism.

The second figure has one base which has six sides. The remaining sides are triangles which meet at a single vertex. It is a hexagonal pyramid.

Subtopic 2 Spheres, Cylinders, and Cones

Expand Their Horizons

In this subtopic, students learn about three solids that are not polyhedra. Spheres, cylinders, and cones are not polyhedra because their surfaces are not made up of polygons; they are curved.

A sphere is the set of all points that are equidistant from a given point. Compare this definition to the definition of a circle. The only difference is that with a circle, the phrase, *in a plane*, is included. By eliminating this phrase, the figure becomes a three-dimensional figure. As with circles, spheres have radii and diameters.

A cylinder has two parallel congruent bases, each of which is a circle. A radius of either base is also a radius of the cylinder. A cone has one circular base and one vertex. As with a cylinder, a radius of the base of a cone is also a radius of the cone. Like prisms, cylinders and cones have altitudes. If the altitude is also the segment joining the centers of the bases or the center of the base to a vertex, it is an axis.

6 Each figure has one curved lateral surface. The first figure has one circular base and one vertex. It is a cone. The second figure has two congruent parallel circular bases. It is a cylinder.

7 A sphere and a cylinder are both solids with curved surfaces, but the sphere does not have any bases, while a cylinder has two bases.

8 The altitude of a cone is a segment that extends from the circular base to its vertex, such that it forms a right angle with the base. The altitude of a cylinder is a segment that joins both bases, and that forms a right angle with both bases. A radius of either figure is the same as a radius of its base.

Additional Examples

1. **Which of the following are not in the shape of a sphere? Explain.**

   - **The moon**
   - **A cereal bowl**
   - **A bicycle rim**

   Neither a cereal bowl nor a bicycle rim is a sphere. A bowl is usually flat on the bottom to keep it from rolling away; also it does not include points above the given center point. A bicycle rim has a circular shape, but it does not include every point equidistant from the center point.

2. **Tell why the figure is neither a cylinder nor a cone.**

   The figure is not a cone because it has two bases and a cone has only one base. The figure is not a cylinder because, while it has two parallel circular bases, the bases are not congruent.

Look Beyond

Students will continue their study of three-dimensional figures in the next lesson which studies nets of prisms. A net is what a solid figure would be like if it were unfolded. Students will also learn how to identify a figure based on its top, side, and front views. Later, students will calculate volume and surface area of some three-dimensional figures.

In higher mathematics classes, students will study oblique prisms, cones, and cylinders. In this lesson, students studied right prisms, right cones, and right cylinders. They were right because the line connecting the center of the base to the vertex was perpendicular to the base or bases. When this segment does not form right angles, the figure is slanted, and it is classified as oblique.

Connections

Some minerals, if not restrained, will grow according to a given structure. For example, quartz crystals form six-sided prisms, and fluorite forms either an octahedral or cubic shape.

Gems are often cut into beautiful symmetrical polyhedral shapes. Look at the stone in a ring or other piece of jewelry and see the different polygonal shapes of the surfaces. Gem-cutting is difficult and precise work, and gem cutters are highly trained in their craft.