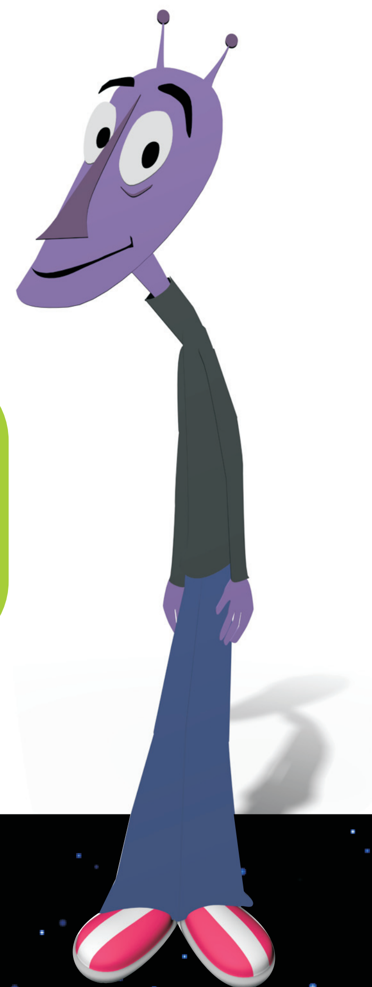
Numbers and Operations

Module 1

Number Sense



Lesson 5
Estimation

Teacher Notes 1.5

Objectives

- Develop and use strategies to estimate the results of whole number computations and to judge the reasonableness of such results.
- Read, write, compare, and solve problems involving whole number computations with and without appropriate technology.

Prerequisites

Adding, subtracting, multiplying, and dividing whole numbers.

Vocabulary

Rounding
Estimate
Compatible numbers
Front-End Estimation

Get Started

- Write the following problem on the board or overhead projector: *Todd is participating in a dance marathon. He wants to raise $100 in pledges. His parents pledged $80; his aunt pledged $70; and his neighbor pledged $40. Does Todd have enough pledges to meet his goal?*
- Ask students to answer the question. **Yes, Todd has enough money.** Probe further by saying, "If the answer is yes, then you must know that the sum of the pledges is greater than $100. Did you actually find the sum of the pledges?" Students may find that they did not need to find $80 + $70 + $40 because their number sense tells them that the sum is greater than $100.
- Explain to students that sometimes, when solving a problem, a general idea of the answer is good enough. In this example, knowing that $80 + $70 + $40 is more than $100 was good enough; the exact sum was not necessary. Say, "When you don't need to find the exact answer to a calculation, you can estimate. In this lesson, we'll learn when an exact answer is needed, and we'll learn a few different estimation methods to use when an estimate will do."

Subtopic 1 — Exact Answer or Estimate

Expand Their Horizons

In Subtopic I, students must determine whether an estimate or an exact answer is required to answer a problem. Sometimes, the problem contains key words such as *about* or *approximately*. These words indicate that an estimated answer will suffice. Other times, however, the problem does not contain key words, and students will have to use their judgment as to whether an exact answer is required. For example, in the Section I example about gasoline, if the student can approximate $2.53 × 3 closely enough to know that the product is greater than $5.00, an exact answer is not necessary.

Before viewing Lesson Notes Problems 1 and 2, remind students that they do not need to answer the question. They simply need to identify whether an estimate is sufficient or whether an exact answer is required.

1 The key word *approximately* indicates that an estimate is sufficient. If time allows, ask students how they might make a quick estimate of the number of fish. (For example, they might count out five fish, then use five as a benchmark to estimate the number of groups of five fish)

2 The key phrase *how many* indicates that an exact answer is needed.

Additional Examples

1. **Estimate or Exact Answer?**
   About how many people can fit in the gymnasium?

   Look for key words or phrases like *about*, *approximately*, or *how many*.

   An estimate is sufficient.

2. **Estimate or Exact Answer?**
   How many people attended Friday night's performance of the school play?

   Look for key words or phrases like *about*, *approximately*, or *how many*.

   An exact answer is needed.

Subtopic 2 — Estimation Strategies: Front-End Estimation

Expand Their Horizons

In Subtopic 2, students add and subtract numbers using Front-End Estimation. With this method, students focus only on the first digit in each number and treat all other digits as if they were zero. Because this procedure might significantly change the number (for example, when 265 is treated as 200), Front-End Estimation is a good strategy when a very rough estimate is all that is required.

Point out that subtraction using Front-End Estimation is not generally a good choice when the minuend and subtrahend are close together (it often leads to differences of zero).

3 Add the digits in the hundreds places of 415 and 748. Since 4 + 7 = 11, a reasonable estimate is 1,100.

4 Subtract the digits in the hundreds places of 570 and 228. Since 5 – 2 = 3, a reasonable estimate of the difference is 300.

Common Error Alert:

Students may be tempted to think ahead and round each number in the problem to its greatest place value. For example, they might round 570 to 600 and find 600 – 200 = 400. While this is a reasonable estimate as well, remind them that Front-End Estimation uses the first digit in the number without rounding it first.

Additional Examples

1. Use Front-End Estimation:

$$\begin{array}{r} 612 \\ 254 \\ \underline{+108} \end{array}$$

Add the first digits of each addend. Ignore the other digits; make them 0.

$$\begin{array}{rcl} 612 & \rightarrow & 600 \\ 254 & \rightarrow & 200 \\ \underline{+108} & \rightarrow & 100 \end{array}$$

$$6 + 2 + 1 = 9$$

$$612 + 254 + 108 \approx 900$$

2. Use Front-End Estimation:

$$\begin{array}{r} 724 \\ \underline{-342} \end{array}$$

Use the first digits to subtract. Ignore the other digits; make them 0.

$$\begin{array}{rcl} 724 & \rightarrow & 700 \\ \underline{-342} & \rightarrow & 300 \end{array}$$

$$7 - 3 = 4$$

$$724 - 342 \approx 400$$

Subtopic 3 Estimation Strategies: Rounding

Expand Their Horizons

In Subtopic 3, rounding techniques are introduced.

Visual learners will appreciate the number line reference given when one of the DVD characters rounds 47 to the nearest 10 by stating that 47 lies closer to 50 than to 40 on a number line. Some students prefer to round by imagining a number line rather than using rounding rules. These students may wonder why, for example, 25 rounds *up to 30* instead of *down to 20* when 25 is equidistant from 20 and 30. Tell students that the practice of rounding fives *up* is nothing more than a convention. In practice, they might sometimes use judgment to round 25 *down to 20*. Students using the rounding rules will notice that of the 10 possible digits in the place to the right of the rounding place, five lead to rounding up and five lead to rounding down.

5 When rounding to the nearest 10 using a number line, have the number line marked by tens (at 0, 10, 20….80, 90, 100). The student must determine whether 92 is closer to 90 or to 100. Using rounding rules, 92 rounds down to 90, since the digit in the one's place is 2.

6 To round to the nearest 100 using a number line, have the number line marked by hundreds (at 0, 100, 200,…800, 900, 1000). Since 995 is closer to 1000 than to 900, it rounds up to 1000. Using rounding rules, 995 rounds up to 1000 because the digit in the ten's place is 9.

Common Error Alert:

Students sometimes lose track of the relevant digits when rounding numbers. Encourage them to develop a system for marking the number to be rounded. For example, they might underline the digit in the rounding place and circle the digit to its right.

Additional Examples

1. Round 648 to the nearest 10:

648 is positioned between 640 and 650. To determine how to round, look at the digit in the ones place. Since 8 > 5, round up.

648 rounded to the nearest 10 is 650.

2. Round 8,234 to the nearest 100:

8,234 is positioned between 8,200 and 8,300. To determine how to round, look at the digit in the ten's place. Since 3 < 5, round down.

8,234 rounded to the nearest 100 is 8,200.

Subtopic 4 Estimation Strategies: Compatible Numbers

Expand Their Horizons

In Subtopic 4, students estimate using compatible numbers. Compatible numbers are numbers that work well together and are easy to use mentally. To estimate 943 ÷ 36, the compatible numbers 900 and 30 are chosen so that the quotient (900 ÷ 30) resembles 9 ÷ 3. Another reasonable choice for this example would be 1000 ÷ 40, which can be easily found using the familiar 100 ÷ 4 = 25. When selecting compatible numbers for addition, students should try to create numbers that group together well, as when 542 + 264 becomes 550 + 250.

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Remind students that when estimating, there is more than one reasonable answer. Some students might select the compatible numbers $340 and $460 in this example.

Additional Examples

1. Estimate using compatible numbers:

$619 + 248$

625 and 250 are close to the original addends and are easily added mentally.

$619 + 248 \approx 875$

2. Estimate using compatible numbers:

$372 \div 58$

Round the problem to 360 ÷ 60 so that it resembles the easily dividable 36 ÷ 6.

$372 \div 58 \approx 6$

Subtopic 5

Comparing and Combining Estimation Strategies

Expand Their Horizons

In Subtopic 5, students can “mix and match” estimation strategies. Remind them that as long as the estimated numbers in the problem stay reasonably close to the original numbers, their estimate is valid. Point out that the closer the rounded numbers are to the original numbers, the closer the estimate will be to the exact answer.

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Because each of the rounded addends is less than the original when 189 + 125 becomes 100 + 100, using pure Front-End Estimation results in an answer that is too low. By rounding each addend to the nearest hundreds, a more accurate estimate is achieved.

Common Error Alert:

Students may be upset when their estimation techniques vary from those used in the DVD and practice problems. Remind them that all the techniques taught in this lesson produce valid estimates. For example, an estimate of $200 in Lesson Notes Problem 8 is too low, but is still a legitimate, rough estimate of the total cost of the items.

Look Beyond

Estimation is necessary to check the reasonableness of arithmetic calculations at every level, and estimating almost always requires rounding the numbers used. As students become more adept at estimating, they will develop techniques to get more accurate results. For example, when adding a series of numbers, they may find that they have rounded most of the numbers up, so they may compensate by rounding selected numbers down.

Connections

Estimation is at the core of everyday consumer math. Consumers must constantly estimate prices, bank balances, payments, and expenses in order to successfully operate within a budget. A consumer might estimate a sum in finding the total price of a meal to ensure there is enough money to pay for it.