Numbers and Operations

Module 1

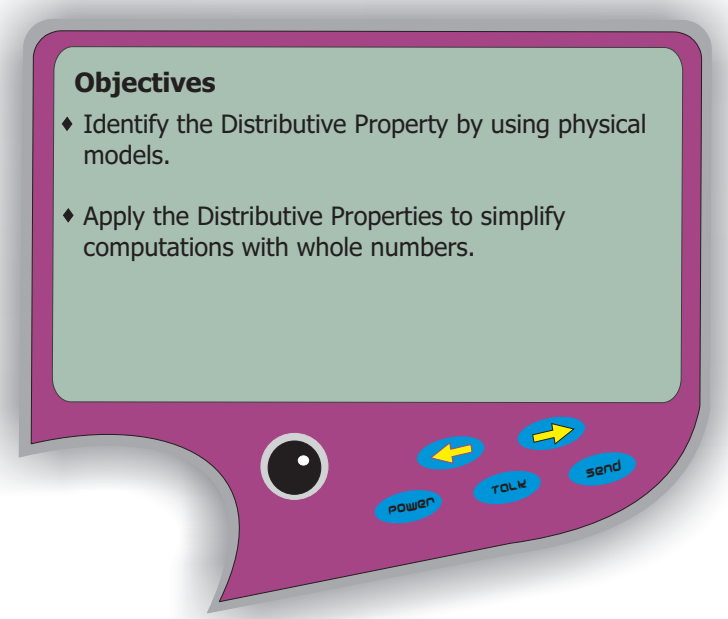
Number Sense

Lesson 4
Distributive Properties

Teacher Notes 1.4

Objectives

- Identify the Distributive Property by using physical models.
- Apply the Distributive Properties to simplify computations with whole numbers.



Prerequisites

Adding, subtracting, multiplying, and dividing whole numbers.

Vocabulary

Distributive Property of Multiplication over Addition

Get Started

- Divide the class into three groups. Ask Group 1 to use counters or grid paper to make an array that models the product 3×6. Ask Group 2 to make an array that models the product 3×7. Ask Group 3 to make an array that models the product 3×13. (It may be necessary to remind students how to model a product with an array.)

- Ask students to name the equations modeled by their arrays. Write their answers on the board as shown:

$$3 \times 6 = 18$$
$$3 \times 7 = 21$$
$$3 \times 13 = 39$$

- Encourage students to look carefully at these three equations and ask them if there is a relationship between the numbers in the first two equations and the numbers in the last equation.
  Yes, each equation has a first factor 3; the second factors equal 13; and the products equal 39.

- Explain that they have just seen an example of the Distributive Property of Multiplication over Addition. This property states that a product, like 3×13,can be found by finding **related or partial** products and adding them together. In this lesson, students will learn how to write and find these products.

Subtopic 1 — Distributive Property Model 1 - Digit Numbers

Expand Their Horizons

In Subtopic 1, students are introduced to the Distributive Property using models. In this section, the property is used to find familiar products, like 4×7. As students watch the instructional portion of the DVD, be sure to point out that the factor that is written as a sum can be written as *any* pair of addends (or any combination of addends, for that matter). For example, 4×7 is written as $4(5 + 2)$ in the DVD but could be written as $4(1 + 6)$ or as $4(3 + 4)$. In the DVD and lesson notes, the sum to be used is specified; in other situations, they are free to choose which sum to use.

Teachers may need to review students on the notation used in this lesson. First, remind students that the symbols $\times$ and $\cdot$ are used interchangeably to denote multiplication, with parentheses being another way to indicate the operation. The expression $2(4 + 3)$ indicates that 2 is to be multiplied by the *entire expression* **(4 + 3)**. Remind them that the parentheses are very important since they distinguish the expression $2(4 + 3)$ from $2(4) + 3$, in which 2 is multiplied by **4**, not by **(4 + 3)**. Point out that the Distributive Property can be applied to simplify $2(4 + 3)$ but not to $2(4) + 3$.

1 Apply the Distributive Property to write $(6 \cdot 3) + (6 \cdot 2)$. Following the Order of Operations, simplify each product inside parentheses first and then add the products.

2 Remind students that they can check their answers by comparing their answer to the basic fact problem shown in the exercise. They know that $3 \cdot 9 = 27$, so they should get 27 when they simplify $3(4 + 5)$ using the Distributive Property.

Common Error Alert:

Students may apply the Distributive Property incorrectly, writing $3(4 + 5) = 3 \times 4 + 5$. Encourage them to check to be sure that the result of the Distributive Property is the sum of two products.

Additional Examples

1. Solve: 8 × 6 = 8(2 + 4)

Apply the Distributive Property. Simplify inside parentheses first and then add. Lastly, check the answer against 8 × 6.

$$\begin{aligned} 8 \times 6 &= 8(2+4) \\ &= (8 \times 2) + (8 \times 4) \\ &= 16 + 32 \\ &= 48 \end{aligned}$$

2. Solve: 4 × 9 = 4(3 + 6)

Apply the Distributive Property. Simplify inside parentheses first and then add. Lastly, check the answer against 4 × 9.

$$\begin{aligned} 4 \times 9 &= 4(3+6) \\ &= (4 \times 3) + (4 \times 6) \\ &= 12 + 24 \\ &= 36 \end{aligned}$$

Subtopic 2

Distributive Property Model 1 - Digit Number Times 2-Digit

Expand Their Horizons

In Subtopic 2, the Distributive Property is used to find the product of a one-digit number and a two-digit number. It is almost always most convenient to write the two-digit factor as the sum of a multiple of ten and a one-digit number. So, 2(14) is written as 2(10 + 4). However, any sum of 14 can be used.

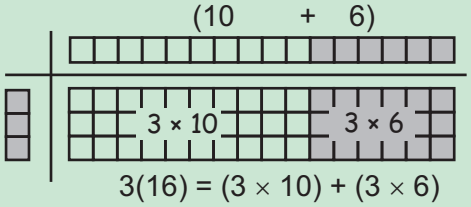
3 The model shows that four rows of 10 and four rows of 2 are equivalent to four rows of 12. So, 4(12) = 4(10 + 2) = (4 · 10) + (4 · 2) = 40 + 8 = 48.

4 Use the Distributive Property to find the partial products 30 and 9. Then, add.

Additional Examples

1. Model: 3(16)

Use an array that shows three rows of 16. Break the array into smaller arrays showing three rows of 10 and three rows of 6.



3(16) = (3 × 10) + (3 × 6)

2. Solve: (5)(18)

Rewrite 18 as 10 + 8. Use the Distributive Property to find the partial products. Then, add the partial products.

$$\begin{aligned} (5)(18) &= 5(10+8) \\ &= 50 + 40 \\ &= 90 \end{aligned}$$

Expand Their Horizons

In Subtopic 3, the Distributive Property is used to find the product of two two-digit numbers. Each of the two factors can be written as the sum of a multiple of ten and a one-digit number. So, (12)(14) is written as (10 + 2)(10 + 4).

5

Point out that each addend in the first factor must be multiplied by each addend in the second factor. So, ten is multiplied by ten and then by four. Then, one is multiplied by ten and four.

Common Error Alert:

When multiplying two sums of this type, students may omit one or more of the partial products. Remind them that there should always be four partial products.

6

Distribute 10 to 10 and 5; distribute 2 to 10 and 5. Add the partial products.

Additional Examples

1. Solve: (16)(13)
(10 + 6)(10 + 3)

Find four partial products by distributing 10 and 6. Find the sum of the partial products.

$$(16)(13) = (10+6)(10+3)$$
$$= 100 + 30 + 60 + 18$$
$$= 208$$

2. Solve: (15)(14)
(10 + 5)(10 + 4)

Find four partial products by distributing 10 and 5. Find the sum of the partial products.

$$(15)(14) = (10+5)(10+4)$$
$$= 100 + 40 + 50 + 20$$
$$= 210$$

Subtopic 4 — Distributive Property of Multiplication Over Subtraction

Expand Their Horizons

In Subtopic 4, students are introduced to the Distributive Property of Multiplication over Subtraction. This rule can be applied when a number is multiplied by a difference.

7 Students may notice that the product 5(17) can be found using the Distributive Property of Multiplication over Addition by writing it as 5(10 + 7). Point out, that in most cases, they will have the choice of using addition or subtraction to rewrite the second factor. Here, however, the use of subtraction for practice purposes is specified.

8 Before having students try this problem, suggest that they leave the factor 12 as is and write 25 as a difference. Some students might also suggest writing (10 + 2)(20 + 5).

Additional Examples

1. Model: 8(26)

Write 26 as 30 – 4. Use the Distributive Property to find 8(30 – 4).

$$\begin{aligned}(8)(26) &= (8)(30-4)\\ &= (8\cdot 30)-(8\cdot 4)\\ &= 240-32\\ &= 208\end{aligned}$$

2. Solve: 15(18)

Write 18 as 20 – 2. Use the Distributive Property to find (15)(20 – 2).

$$\begin{aligned}(15)(18) &= (15)(20-2)\\ &= (15\cdot 20)-(15\cdot 2)\\ &= 300-30\\ &= 270\end{aligned}$$

Subtopic 5 — Applications of the Distributive Property

Expand Their Horizons

In Subtopic 5, students solve word problems using the Distributive Property. Note that in the problem on the DVD and in the practice problems, it is not specified whether to use Addition or Subtraction. Choices could vary among students. Use the opportunity to show that the answer is the same regardless of how the factors are rewritten.

9 Some students might write 52(30 + 3), then attempt to distribute. While this strategy is valid, it does not lend itself to mental math as well as the option used in the DVD.

Additional Examples

1. Luria has 7 space stone necklaces. Each necklace has 28 space stones. How many space stones does Luria have?

Find 7(28) by writing 7(30 – 2), then distributing.

$$\begin{aligned}(7)(28) &= (7)(30-2)\\ &= (7\cdot 30)-(7\cdot 2)\\ &= 210-14\\ &= 196\end{aligned}$$

Luria has 196 space stones.

2. A space shuttle can fly 28 light years on one kiloblong of fuel. How far can the space shuttle fly on 32 kiloblongs of fuel?

Find (28)(32) by writing (20 + 8)(30 + 2), then distributing. First, distribute 20. Next, distribute 8. Find the partial products and then add.

$$\begin{aligned}(28)(32) &= (20+8)(30+2)\\ &= 20(30+2)+8(30+2)\\ &= 20\cdot 30+20\cdot 2+8\cdot 30+8\cdot 2\\ &= 600+40+240+16\\ &= 896\end{aligned}$$

The space shuttle can fly 896 light years on 32 kiloblongs of fuel.

Look Beyond

The Distributive Property is used extensively in algebra. When multiplying binomials, such as (x + 3)(x + 2), you will have four partial products, $x^2 + 2x + 3x + 6$, the sum of which would be $x^2 + 5x + 6$. Likewise when factoring a polynomial, such as $x^2 + 5x + 6$, the student must look for four partial products that when added together will give the total product.

In addition, the Distributive Property can be used to show $3(x – 4) = 3x – 12$. Note that since $3(x – 4) = 3x – 12$; it is also true that $3x – 12 = 3(x – 4)$. This application of the Symmetry Property of Equality means that the Distributive Property can be used not only to multiply factors but to *factor products*. So, the equation $3x – 12 = 0$ can be written as $3(x – 4) = 0$ using the Distributive Property. When one side of an equation is written as a product, and the other is zero, the Zero Product Property can be used to solve the equation. Because of the Zero Product Property, one of the factors (3 or $x – 4$) must be zero. Since three can not be zero, it must be true that $x – 4 = 0$, so the solution of the equation is $x = 4$.

Connections

The Distributive Property can be useful in calculating products mentally. In this lesson, it was shown how to find the product of a one-digit number with another one-digit number and with a two-digit number. However, it can be used to find the product of any two numbers. For example, to find the total number of hours worked during 259 eight-hour workdays, we need to find 8 × 259. Using the Distributive Property.

$$\begin{aligned}8(259) &= 8(200 + 50 + 9)\\ &= 8(200) + 8(50) + 8(9)\\ &= 1{,}600 + 400 + 72\\ &= 2{,}072.\end{aligned}$$