Numbers and Operations

★ Module 1 ★

Number Sense

Lesson 3
Properties of Addition and Multiplication and Inverse Operations

Teacher Notes 1.3

Objectives

- Identify properties of addition and multiplication: Commutative, Associative, Identity, and Multiplicative Property of Zero.
- Apply the properties of addition and multiplication to simplify computations with whole numbers and to solve problems using mental math.

Prerequisites

Adding, subtracting, multiplying, and dividing whole numbers.

Vocabulary

- Commutative Property of Addition/Multiplication
- Associative Property of Addition/Multiplication
- Identity Property of Addition/Multiplication
- Multiplicative Property of Zero
- Mental math

Get Started

- Write 14 + 32 + 6 + 8 on the board and ask students to evaluate the expression by finding the sum.
- Ask for a volunteer to explain his or her process in finding the sum. Some students may simply add the addends as they appear from left to right. If so, ask, “Did anyone use a different strategy to find the sum? Did anyone add the addends in a different order? Did anyone make groups?” Find a student who evaluated (14 + 6) + (32 + 8) = 20 + 40 = 60.
- Ask, “How do you know you can write the addends in a different order? How do you know you can group the addends? In this lesson, you will learn the names of several properties of addition and multiplication. These properties make mental math easier.”

Expand Their Horizons

In Subtopic 1, the Commutative, Associative, and Identity Properties of Addition are introduced. Students should have an intuitive understanding of these properties but may have trouble associating the names with the properties. Help them understand the names by focusing on the related words of *commute, associate, and identical*.

When a parent *commutes* to work, he or she *goes back and forth*, changing the direction of travel and the order of their destinations. Similarly, the addends in a commutative statement change direction or order. When two or more people *associate*, they “hang out” together in a group. Similarly, addends in an associative statement are grouped. To remember the Identity Property of Addition, students can ask themselves, “What can I add to a number so that the sum is **identical** to the original number?”

Make sure students understand what the word *property* means. A property is a fact that can be shown to be true and does not change. A property of water states that water boils under normal circumstances at 212°F. A property of squares states that all four sides in a square have the same length. Say to students, “Imagine if the properties of addition did not exist. What if 2 + 3 were not guaranteed to be equal to 3 + 2? In order to use math effectively, we must have certain rules. The properties state these rules and assure us that we can do things such as change the order of addends.”

Common Error Alert:

Some students may incorrectly assume that the Associative and Commutative Properties also apply to subtraction and division. Show them that $18 - 6 \neq 6 - 18$ and $(18 \div 6) \div 3 \neq 18 \div (6 \div 3)$. These two properties only apply to addition and multiplication.

1 Emphasize that the equal sign separates expressions, which have the same value. So, 12 + 35 = 35 + 12 shows two expressions, each equal to 47. Since the order of the addends changes, this equation shows the Commutative Property of Addition.

2 Point out that the order of addends does not change, only the grouping changes. The expression shows the Associative Property of Addition.

3 The order of the addends 5 and (8 + 4) changes, but the grouping does not. The expression shows the Commutative Property of Addition.

Common Error Alert:

Students may see the parentheses in this equation and think it shows the Associative Property of Addition. Point out that there are two addends: 5 and the expression (8 + 4). The order of these two addends is different on the left side of the equation than it is on the right side, but the grouping did not change.

4 The equation 3 + 0 = 3 shows the Identity Property of Addition. Zero is added to three, and the result is three.

Additional Example

1. **Name the property shown:**
   4 + (3 + 7) = (4 + 3) + 7

   The order of the addends does not change.

   Since the grouping changes, this equation shows the Associative Property of Addition.

2. **Name the property shown:**
   10 + (4 + 0) = 10 + 4

   Remind students to study each side of the equation. On the left, the addends are 10 and (4 + 0); on the right, the addends are 10 and 4.

   The equations show the Identity Property of Addition, which states that 4 + 0 = 4.

Subtopic 2 — Mental Math Using Properties of Addition

Expand Their Horizons

In Subtopic 2, students use Properties of Addition to simplify expressions using mental math. To *simplify* an expression is to write the expression in its simplest form by carrying out the indicated operations. Encourage students to try the examples and exercises without any pencil-and-paper computation.

5 When simplifying addition expressions, it is usually most convenient to find a group (or groups) of addends that have a sum that is a multiple of 10 or 100 (since these multiples have a zero in the ones place, they are easy addends to work with mentally). So, in this expression, it will be advantageous to add 96 and 4 first. Note that 96 and 24 also make a good pair, but finding 96 + 24 is more difficult than finding 96 + 4.

Common Error Alert:

Students may be confused by the use of two properties of addition in this example. Point out that the Commutative Property is used to change the order of the addends; then, the Associative Property is used to group the addends.

Additional Examples

1. **Simplify using mental math:**
   12 + 25 + 38

   Look for a pair of addends for which the sum is a multiple of 10 or 100.

   12 + 25 + 38
   12 + 38 + 25
   (12 + 38) + 25
   50 + 25
   75

2. **Simplify using mental math:**
   22 + 13 + 38 + 17

   Look for a pair of addends for which the sum is a multiple of 10 or 100.

   22 + 13 + 38 + 7
   22 + 38 + 13 + 7
   (22 + 38) + (13 + 7)
   60 + 20
   80

Subtopic 3 — Properties of Multiplication

Expand Their Horizons

In Subtopic 3, the Commutative, Associative, and Identity Properties of Multiplication are introduced, along with the Multiplicative Property of Zero. The first two of these properties mirror their additive counterparts, stating that the order and grouping of multiplication does not change the final result.

Compare and contrast the Identity Properties of Addition and Multiplication and the Multiplicative Property of Zero. Point out to students that the Identity Property of Multiplication contains a one, not a zero, since multiplying a number by one produces a result identical to the original number (3 + **0** = 3, but 3 × **1** = 3). The Multiplicative Property of Zero states that the product of any number and zero is zero (3 + 0 = **3**, but 3 × 0 = **0**).

6 Since the order of the factors changes, the equation shows the Commutative Property of Multiplication.

7 The equation shows that the product of a number and zero is equal to zero. The equation demonstrates the Multiplicative Property of Zero.

8 The left side of the equation contains two factors: 7 and (6 × 5). The right side of the equation shows the product of these factors in another order. The equation shows the Commutative Property of Multiplication.

Common Error Alert:

Students may assume that since the position of the parentheses changes from the left side to the right side, the equation shows the Associative Property of Multiplication. Encourage them to think of the expression (6 × 5) as a single factor. Point out that the expression inside parentheses does not change; the parentheses and its contents simply switched places with the factor seven. If students have trouble seeing this, encourage them to read the expression as "seven times parentheses is equal to parentheses times seven." This auditory clue will help them see that the equation shows the Commutative Property of Multiplication.

9 The left side of the equation contains three factors: 3, 2, and 8. The right side of the equation shows the product of these factors in the same order but grouped differently. The equation shows the Associative Property of Addition.

10 This equation shows that the product of a number and one is equal to the number. The equation demonstrates the Identity Property of Multiplication.

Subtopic 4 — Mental Math Using Properties of Multiplication

Expand Their Horizons

In Subtopic 4, students simplify multiplication expressions using Properties of Multiplication.

11 In this expression, students will have to create compatible factors by writing one of the factors as a product. Remind them that the goal is to create factors where the product will be a factor that is easy to multiply mentally. Since 25 × 4 = 100, write 36 as 4 × 9 and multiply 25 × 4 × 9. Explain that while they could rewrite 36 as 12 × 3 or 18 × 2, there would be no advantage with these choices.

Additional Examples

1. Simplify: 50 × 38

Since 50 × 2 = 100, write 38 as the product of 2 and another number.

50 × 38
50 × 2 × 19
100 × 19
1,900

2. Simplify: 25 × 9 × 4 × 5

Use the Commutative Property to write the factors in a different order and then use the Associative Property to group the factors.

25 × 9 × 4 × 5
25 × 4 × 9 × 5
(25 × 4) × (9 × 5)
100 × 45
4,500

Look Beyond

Understanding properties of equality as applied to whole number operations is a building block for algebra. Students will use the properties to simplify expressions containing variables. For example, the expression $a(ba)$ is equivalent to $a(ab)$ by the Commutative Property of Multiplication; $a(ab)$ is equivalent to $(aa)b$ by the Associative Property; and the expression can be written in simplified form as a^2b. In some courses, students will use properties of real numbers as justifications in the steps of formal algebraic proofs. In advanced mathematics courses, the properties can be proven using the Principle of Mathematical Induction.

Connections

Properties of Equality are used so commonly it is easy to take them for granted and to pass them off as esoteric math theory. Encourage students to see examples of the properties in the world around them. For example, bring the properties to life by saying, "There were 23 students in the room at the beginning of class. No more students have entered the room; therefore, there are still 23 students in the room. We have demonstrated the Identity Property of Addition. Let's look at another example. Joe put a nickel followed by a quarter into his left pocket. He then placed a quarter followed by a nickel into his right pocket. Each pocket contains 30¢. The order in which the coins were added to Joe's pockets does not matter. This shows the Commutative Property of Addition." Ask students to think of other real-world scenarios that demonstrate the properties.