Numbers and Operations

Module 1

Number Sense

Lesson 2
Divisibility Rules

Teacher Notes 1.2

Objectives

- Use divisibility rules to determine if a number is a factor of another number (2,3,4,5,6,9, and 10).

Vocabulary

Divisible
Factor
Digit
Remainder
Even Numbers

Prerequisites

Adding, subtracting, multiplying, and dividing whole numbers

Get Started

- Give each student or group of students 24 counters.
- Ask the students to divide the counters into groups with the same number of counters in each group. Tell them to be sure to use every counter.
- Ask students to describe their groupings.
  Possible answers: two groups of 12, three groups of 8, four groups of 6, etc.
- Say, "You can divide 24 into three equal groups because 24 is divisible by 3. There are no counters left over. That means there is no remainder. Dividing 3 into 24 is easy, but what if I had given you 398 counters? Finding equal groups would have been harder. In this lesson you will learn a rule to tell whether a number is divisible by 3. You will also learn rules that will let you know whether a number is divisible by 2, 4, 5, 6, 9 or 10."

Expand Their Horizons

Divisibility rules for 2, 5, and 10 are presented in Subtopic 2. Divisibility by 4 is presented in Subtopic 3. Before watching the DVD, be sure students understand what *divisible* means. Remind them that a number is divisible by another if the number can be divided into equal groups with nothing left over. (This is a less formal version of the definition given in the lesson.)

Students most likely have the number sense to understand the divisibility rules for 2, 5, and 10 and may even offer them before watching the DVD. These rules should come easily to them.

The divisibility rule for 4 may be more troublesome for students. Explain how the rule works to help ease their confusion. Ask students to think of the number 312. Remind them that the number can be written as 300 + 12, and that this expression can be modeled with one group of 300 counters and another group of 12 counters. The group of 300 counters can be put into four equal groups of 75 each, and the group of 12 counters can be put into four equal groups of 3 each. If the two sets of groups are merged, there will be four equal groups of 78 counters each, so 312 is divisible by 4.

This model can be used to test any whole number greater than 100 for divisibility by 4. The key to understanding the divisibility rule for 4 lies in the fact that *any multiple of 100 is divisible by 4* (since each 100 is made up of 4 groups of 25). So, 300 is divisible by 4, just as 800; 1,500; 368,590,200; and all other multiples of 100 are divisible by 4. When a number is written as the sum of a multiple of 100 and a two-digit number (like 300 + 12), it is only the 2-digit number that we must test for divisibility by 4; we already know the first addend is divisible by 4.

1 Since 546 ends in 6, it is divisible by 2, but not by 5 and 10. Since 46 ÷ 4 is not a whole number, 546 is not divisible by 4.

Common Error Alert:

Students may misinterpret the direction line. Remind students that they should test the given number for divisibility by each of the numbers 2, 5, and 10. Be sure they understand that a number can be divisible by more than one number.

2 Since 430 ends in 0, it is divisible by 2, 5, and 10. Since 30 ÷ 4 does not divide evenly, 430 is not divisible by 4.

Common Error Alert:

Some divisibility rules require the student to look at the last digit in the number; others require them to look at the last *two* digits. Students may become confused by this distinction. For example, they might think that 636 is not divisible by 4 because the last digit is not 4. Encourage students to make a chart or list of the divisibility rules and to study them to avoid confusion.

3 Since 425 ends in 5, it is divisible by 5, but not by 2 and 10. Since 25 ÷ 4 does not divide evenly, the quotient is not a whole number; so, 425 is not divisible by 4.

4 Since 636 ends in 6, it is divisible by 2, but not by 5 and 10. Since 36 ÷ 4 = 9, 636 is divisible by 4.

Additional Examples

1. **Is 238 divisible by 2, 4, 5, or 10?**

   To test for divisibility by 2, 5, and 10, look at the digit in the ones place (8).
   To test for divisibility by 4, look at the last two digits in the number (38).

   238 is divisible by 2.
   238 is not divisible by 4.
   238 is not divisible by 5.
   238 is not divisible by 10.

2. **Is 650 divisible by 2, 4, 5, or 10?**

   To test for divisibility by 2, 5, and 10, look at the digit in the ones place (0).
   To test for divisibility by 4, look at the last two digits in the number (50).

   650 is divisible by 2.
   650 is not divisible by 4.
   650 is divisible by 5.
   650 is divisible by 10.

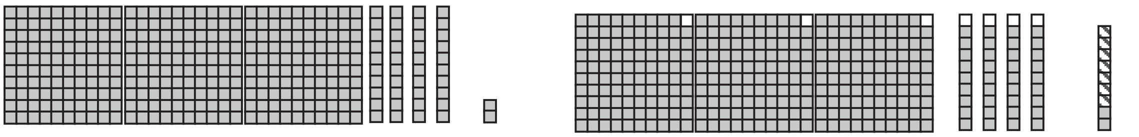
Subtopics 4 & 5 — Divisibility by 3, 6 and 9

Expand Their Horizons

This section introduces the divisibility tests for 3 and 9.

Understanding these divisibility rules is more difficult. The DVD presents the rule but does not explain its derivation, which follows here:

To test a number like 342 for divisibility by 3, think of the number as 3 hundreds, 4 tens, and 2 ones. We know that both 99 and 9 are divisible by 3, so we take **one** away from each group of 100 and each group of 10. Our number is now modeled as three groups of 99, four groups of 9, 2 ones, and 7 ones (the additional ones were removed from the hundreds and tens). The original and remodeled numbers are shown in the diagram below. The groups of 99 and 9 are certainly divisible by 3; the important question is *whether the set of ones* is divisible by 3. Notice that since one unit was taken away from each digit before the ones place, the sum of the digits, 3 + 4 + 2, represents the number of ones in the remodeled number. It is *that number* which must be divisible by 3 in order for 342 to be divisible by 3. A similar argument shows the divisibility rule for 9.



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Since 876 has 6 in its ones place, it is divisible by 2 but not by 5 or 10. Since 76 ÷ 4 = 19, 876 is divisible by 4. Since 8 + 7 + 6 = 21 and 21 is divisible by 3, 876 is divisible by 3. Since 21 is *not* divisible by 9,876 is not divisible by 9. Since 876 is divisible by both 2 and 3, it is divisible by 6.

Additional Examples

1. **Is 387 divisible by 2, 3, 4, 5, 6, 9, or 10?**

   To test for 2, 5, and 10, look at the digit in the ones place (7).
   To test for 4, look at the number formed by the last two digits (87).
   To test for divisibility for 3 and 9, find the sum of the digits (3 + 8 + 7 = 18).
   To test for divisibility by 6, determine whether 387 is divisible by 2 and 3.

   387 is divisible by 3 and 9.
   387 is not divisible by 2, 4, 5, 6, or 10.

2. **Is 584 divisible by 2, 3, 4, 5, 6, 9, or 10?**

   To test for 2, 5, and 10, look at the digit in the ones place (4).
   To test for 4, look at the number formed by the last two digits (84).
   To test for divisibility for 3 and 9, find the sum of the digits (5 + 8 + 4 = 17).
   To test for divisibility by 6, determine whether 587 is divisible by 2 and 3.

   584 is divisible by 2 and 4.
   584 is not divisible by 3, 5, 6, 9, or 10.

Look Beyond

When operating on algebraic expressions, students will often need to test for divisibility. Writing expressions in factored form (i.e. as the product of two or more other expressions) is an important way to simplify complicated expressions and to solve equations. In order to perform these procedures quickly, it will be helpful for the mathematics student to have these divisibility rules in their bag of tricks. For example, the divisibility rule for 9 comes in handy when writing the fraction $\frac{549}{694}$ in simplest form. Testing the numerator and the denominator for common factors is the first step in writing a fraction in simplest form. A divisibility test shows that both numerator and denominator are divisible by 9. When the fraction is written as $\frac{9 \cdot 61}{9 \cdot 78}$, it can be simplified to the fraction $\frac{61}{78}$.

Connections

Division is a basic operation with many applications in everyday life. The rules students learn to facilitate mental math are very useful. They might use divisibility rules to determine whether a pack of candies can be divided equally among three friends, whether an item can be paid for using only $10 bills, or whether a group of athletes can be divided into six teams of equal size before a tournament.