Numbers and Operations

Module 1

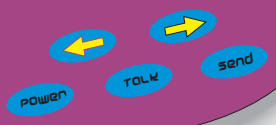
Number Sense

Lesson 1
Order of Operations

Teacher Notes 1.1

Objectives

- Apply rules for Order of Operations to equations with whole numbers and parentheses.
- Apply rules for Order of Operations to equations with whole numbers and with or without parentheses, brackets, or exponents.
- Apply rules for Order of Operations to rational numbers.



Prerequisites

Adding, subtracting, multiplying, and dividing whole numbers.

Vocabulary

Expression
Evaluate
Addition
Subtraction
Multiplication
Division
Grouping symbols
Brackets
Exponents
Parentheses
Base

Get Started

- Write the following list of operations on the board:
  - divide by 5
  - add 5
  - multiply by 2.

- Have students write an equation that begins with the number 25 and includes each of the listed operations. They may place the operations in any order but must use each exactly once.

- Ask, "In how many different orders can you perform the operations?" **(6 ways)** Ask the students to share their results.

[(25 ÷ 5) + 5] × 2 = 20 **[(25 × 2) ÷ 5] + 5 = 15** **[(25 + 5) × 2] ÷ 5 = 12**
[(25 ÷ 5) × 2] + 5 = 15 **[(25 × 2) + 5] ÷ 5 = 11** **[(25 + 5) ÷ 5] × 2 = 12**

- Ask, “Why are there so many different answers?” Point out that the order in which the operations are performed affects the final answer.

- Say, “When you see an expression containing more than one operation, you must know the proper order in which to perform the operations. This lesson will introduce you to the proper Order of Operations.”

Subtopics 1 & 2

Order of Operations (PEMDAS) and Problems Involving Operations

Expand Their Horizons

Throughout this lesson, be sure to use the terms “evaluate” and “expression” liberally. Remind students that an arithmetic expression is a mathematical phrase that contains a number or numbers and operations. Unless it is a single number (e.g. 3, 12, -9), an expression contains an operation that shows *work to be done*. When every operation in an expression has been done, the expression has been simplified to one number. To *evaluate* an expression means to complete all operations that the expression indicates and to find its single-number equivalent. Remind students that they have been evaluating simple expressions like 1 + 1, 5 × 3 and 3^2 for a long time.

1

Before students simplify expressions, have them number the order of operations:

$$20 - 7 \times 2 + 1.$$

As they simplify expressions in this lesson, encourage students to perform one operation at a time. As each operation is carried out, they should re-copy the problem on a new line, substitute the simplified value, and write all other parts of the problem as they appeared in the previous line. For example, their first line of work in this problem should show 20 – 14 + 1.

Common Error Alert:

Students may interpret the acronym PEMDAS incorrectly. They may assume that since “M” precedes “D” (and “A” precedes “S”) in the acronym, that multiplication will precede division (and addition will precede subtraction). They may find products before quotients (and sums before differences) instead of treating multiplication and division (and addition and subtraction) as equals and performing the operations as they appear from left to right. Teachers may want to write P E M D A S to stress that multiplication and division are performed as they appear from left to right. Likewise, addition and subtraction are performed as they appear from left to right.

2

Before beginning to evaluate the expression, ask students to verbalize the order in which they will perform the operations. In the expression 14 + 6 ÷ 2 × 3, the order will be *divide, then multiply, then add*.

Additional Examples

1. Evaluate: 16 – 8 ÷ 2 + 5

Divide, then subtract, then add.

$$16 - \overset{2}{8} \div \overset{1}{2} + \overset{3}{5}$$
$$16 - 4 + 5$$
$$12 + 5$$
$$17$$

2. 5 × 4 ÷ 2 + 5

Multiply, then divide, then add.

$$\overset{1}{5} \times \overset{2}{4} \div \overset{3}{2} + 5$$
$$20 \div 2 + 5$$
$$10 + 5$$
$$15$$

Subtopic 3 — Order of Operations Involving Parentheses

Expand Their Horizons

This section introduces expressions containing parentheses.

Grouping symbols may be used to alter the standard order of operations. For example, the total amount paid for $7 admission tickets for 3 adults and 12 students at an amusement park could be found using the expression (3 + 12) × 7. To evaluate this expression, add first and then multiply.

3 Remind students that they may remove a grouping symbol when it contains no operations. For example, the next line in the solution of 8 + 2 × (5 – 3) can be written as 8 + 2 × 2 instead of 8 + 2 × (2).

4 Highlight the effect of the use of parentheses in this expression by having students evaluate the expression written without parentheses, 56 – 4 + 1 × 6.

Additional Examples

1. 60 - (4 + 3) x 8

Evaluate 4 + 3 first, then multiply, then subtract.

$$60 - \overset{3}{(}\overset{1}{4} + 3\overset{)}{)} \times \overset{2}{8}$$
$$60 - 7 \times 8$$
$$60 - 56$$
$$4$$

2. 15 + 3 x (2 + 4)

Evaluate 2 + 4 first, then multiply, then add.

$$15 + \overset{3}{3} \times \overset{2}{(}\overset{1}{2} + 4\overset{)}{)}$$
$$15 + 3 \times 6$$
$$15 + 18$$
$$33$$

Expand Their Horizons

This section introduces expressions containing nested grouping symbols and expressions containing exponents.

If students have difficulty focusing on the innermost set of grouping symbols, instruct them to highlight each pair of grouping symbols in a different color and then to look for the pair that has no other pairs within it.

Tell students that the decision about which expression to evaluate first does not depend on the *type* of grouping symbol used but rather on the position of the grouping symbols. It is most common that parentheses are used first; then, if another grouping is needed, brackets are used. If a third pair is required, it is usually braces. Point out that this order is a convention rather than a rule and provide examples that show different combinations of parentheses, brackets, and braces. For example, direct students to evaluate $3+(4\times[3+1])$ and $[8-\{4\div 2\}]-4$.

5 Remind students that when a number lies immediately before a set of parentheses with no operation symbol separating it from the parentheses, multiplication is indicated. In 2[4(6 – 3)] – 10, 4 is multiplied by the expression (6 – 3), and 2 is multiplied by the expression [4(6 – 3)]. In order to show that multiplication is indicated, advise students to retain the parentheses after evaluating 6 – 3, so that the expression reads 2[4(3)] – 10. For the same reason, instruct students to retain brackets in the next step: 2[12] – 10.

6 Remind students that 2^3 represents the product $2 \times 2 \times 2$.

Common Error Alert:

Students may be careless when evaluating exponents. For example, they may write $2^3 = 6$ instead of $2^3 = 8$. Remind them to be careful not to confuse multiplication with exponents.

Additional Examples

1. 7 + [20 – 3(2 x 3)]

Evaluate the expression in parentheses first. Then, use *Order of Operations* to evaluate the expression inside brackets.

$$7+[20-3(2\times 3)]$$
$$7+[20-3(6)]$$
$$7+[20-18]$$
$$7+2$$
$$9$$

2. $4 + (6 \times 3) \div 3^2$

Evaluate (6×3) first, then 3^2, then divide, and then add.

$$4+(6\times 3)\div 3^2$$
$$4+18\div 3^2$$
$$4+18\div 9$$
$$4+2$$
$$6$$

Look Beyond

The use of the Order of Operations is fundamental to every future algebraic endeavor students undertake. In future courses, students will write algebraic expressions (expressions containing variables, like $3x$, $2xy^2$, and $4a - 2b$). Understanding the Order of Operations in arithmetic expressions is essential to writing, understanding, and evaluating algebraic expressions.

Calculator

Students often evaluate numerical expressions using a calculator. If available, show them how to enter an expression so that the operations are carried out in the intended order. Some basic four-function calculators perform operations in the order in which they are entered and some do not. When evaluating 3 + 2 × 5, entering [3] [+] [2] [×] [5] will result in [25]. In order to achieve the desired result, the expression could be entered as [2] [×] [5] [+] [3].

Scientific calculators and graphing calculators have the ability to perform operations according to the *Order of Operations*. These calculators often have keys for parentheses which enables one expression to be evaluated before another operation is carried out: [3] [+] [(] [2] [×] [5] [)].

Graphing calculators allow an entire expression to be entered before it is evaluated, and the calculator obeys the order of operations: