

NAME _____

Module 2 Whole Number Operations
Lesson 5 Problem-solving Strategies

Challenge Problems

2.5

Set 1

- 1 What number is in the ones place of 3^{54} ?
- 2 Find the sum of the integers from one to 200.
- 3 A space camp farmyard has only unicorns and ostriches. There are 65 animals and 192 legs. How many unicorns are there in the farmyard?
- 4 A survey of 100 students found that 45 run, 30 swim, 20 lift weights, six run and swim, five swim and lift weights, one runs and lifts weights, and one does all three. How many students only lift weights?

Possible Answers:

1. **Look for a pattern. List the powers of three, each time paying attention to the digit in the ones place. The pattern of ones places has four repeating terms: three, nine, seven, and one. 54 divided by four is 13, remainder two. That means the pattern repeats 13 times and three to the 54th power is two terms later. The last digit of the every second term in the pattern is nine.**

$$3^1 = 3 \qquad 3^5 = 243$$

$$3^2 = 9 \qquad 3^6 = 729$$

$$3^3 = 27 \qquad 3^7 = 2,187$$

$$3^4 = 81 \qquad 3^8 = 6,561$$

$$54 \div 4 = 13 \text{ R}2$$

Nine is in the ones place of 3^{54} .

2. Look for a pattern. Write all the addends from one to 100, and then write the rest of the addends underneath, in reverse order. Then, start adding pairs of numbers. Each sum is 201. There are 100 such pairs. $100 \times 201 = 20,100$. The sum of the numbers from one to 200 is 20,100.

$$\begin{array}{r}
 1 + 2 + 3 + \dots + 99 + 100 \\
 \underline{200 + 199 + 198 + \dots + 102 + 101} \\
 201 + 201 + 201 + \dots + 201 + 201
 \end{array}$$

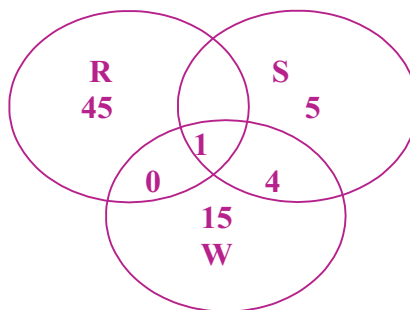
$$\begin{array}{r}
 100 \times 201 = 20,100 \\
 20,100
 \end{array}$$

3. Guess and check. Make a table to record the number of unicorns, the number of ostriches, the number of unicorn legs, the number of ostrich legs, and the total number of legs. The number of unicorns plus the number of ostriches has to be 65; as a first guess, use 25 and 40. That gives a total of 180 legs, which is too low. Increase the number of unicorns to 35. Now there are 200 total legs, which is too high. Thirty unicorns and 35 ostriches are too low. Thirty-one unicorns and 34 ostriches give the right total of legs in the farmyard.

| U | O | U legs | O legs | Total |
|----|----|--------|--------|-------|
| 25 | 40 | 100 | 80 | 180 |
| 35 | 30 | 140 | 60 | 200 |
| 30 | 35 | 120 | 70 | 190 |
| 31 | 34 | 124 | 68 | 192 ✓ |

31 unicorns
34 ostriches

4. Make a diagram.



15 students only lift weights.