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## Challenge Problems

Lesson 4 Three-Dimensional Shapes

## Set 1

1) Euler's Theorem, $(F+V=E+2)$, states that for any polyhedron, the number of faces $F$ plus the number of vertices $V$ equals the number of edges $E$ plus two. Verify that the formula is valid for each Platonic solid.

(2) A polyhedron has 14 faces: six octagons and eight triangles. How many vertices does it have?


## Set 2

(1) Compare prisms and cylinders. How are they alike? How are they different?
(2) Compare cones and pyramids. How are they alike? How are they different?

Module 10 Coordinate Geometry and Spatial Visualization
Lesson 4 Three-Dimensional Shapes

## Possible Answers

Set 1

1. Solid: $\quad F+V=E+2$

Tetrahedron: $\mathbf{4 + 4 = 6 + 2}$
Octahedron: $8+6=12+2$
2. Each octagon has eight edges. Since there are six octagons, multiply six times eight edges. Each triangle has three edges. There are eight triangles, so that is eight times three edges. That is 72 edges in all. Every edge was counted twice because every edge is shared by exactly two faces. There are really only 36 edges. Substitute $\mathbf{1 4}$ for $F$ and $\mathbf{3 6}$ for $E$ in Euler's Formula and solve for $V$. There are 24 vertices.

$$
\begin{gathered}
(6)(8)+(8)(3)=72 \\
72 \div 2=36 \text { edges } \\
F+V=E+2 \\
14+V=36+2 \\
14+V=38 \\
V=24 \\
24 \text { vertices }
\end{gathered}
$$

Set 2

1. Prisms and cylinders are both solids, and they both have two bases. However, a prism is a polyhedron, so its bases are polygons. A cylinder is not a polyhedron. Its bases are circles.
2. Cones and pyramids are both solids, and they both have one base. However, a cone is not a polyhedron because it has curved surfaces. Its base is a circle. A pyramid is a polyhedron. Its base is a polygon.
