NAME

Module 18   Solving Radical Equations
Lesson 4   Solving Problems Using the Distance
              and Midpoint Formulas

independent practice

Round distances to the nearest hundredths.

1. Find the approximate distance between points $P(-4, 1)$ and $Q(3, -8)$. Then, find the midpoint of $\overline{PQ}$.

PQ: 11.40 units; M: $\left(-\frac{1}{2}, -\frac{7}{2}\right)$

2. Find the approximate distance between points $A(6, -1)$ and $B(-2, -3)$. Then, find the midpoint of $\overline{AB}$.

AB: 8.25 units; M: (2, -2)

3. Sonia is having her hair done at a salon located three miles west and four miles north of school. Lauren is having her nails painted at a salon located six miles west and eight miles south of school. Approximately how far apart are the girls?

12.37 miles

4. George is standing three blocks west and eight blocks north of the library. His car is parked seven blocks east and two blocks south of the library. What is the approximate distance between George and his car?

14.14 blocks

5. An ornithologist tracked a tagged bird flying from a point six miles west and eight miles south of a pond, to a point 10 miles west and 13 miles south of the same pond. Assuming the bird flew in a straight line between the two locations, approximately how far did the bird fly?

6.40 miles

6. Harry's house is located at a point halfway between his aunt's house and his grandmother's house. If his aunt lives seven miles east and one mile south of Harry's school and his grandmother lives seven miles east and five miles north of Harry's school, what is the location of Harry's house, relative to his school?

Harry's house is located seven miles east and two miles north of his school.

7. What is the location of the center of a circle if one of its diameters has endpoints at (3, -5) and (4, 3)?

$\left(\frac{7}{2}, -1\right)$

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8. a. Stacey and Ned work in the same office building. Stacey lives four miles north of the building, and Ned lives four miles east and two miles south of the building. What is the approximate distance between their homes?

7.21 miles

b. Roger lives at a point halfway between Stacey and Ned. Approximately how far does Roger live from the office building?

2.24 miles

9. a. A stranded hiker radios for help, reporting that he is injured at a location 12 km east and four km south of a large pond. A rescue helicopter is located 10 km west and two km north of the pond. If the helicopter flies directly to the hiker, approximately how far will it travel?

22.80 kilometers

b. Relative to the pond, what is the helicopter's location when it is halfway to the hiker?

The helicopter is one mile east and one mile south of the heliport.

Journal

1. Explain why $\sqrt{(|x_2 - x_1|)^2 + (|y_2 - y_1|)^2}$ can be written as $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. To find the approximate distance between the points $(3, -2)$ and $(1, 4)$, Sarah made the following calculation.
$d = \sqrt{(3 - (-2))^2 + (1 - 4)^2} = \sqrt{(5)^2 + (-3)^2} = \sqrt{25 + 9} = \sqrt{34}$
Sarah's work is incorrect. Identify her mistake and find the correct distance.
3. Two students found the distance from $(5, -2)$ to $(3, 1)$. Their first steps are shown below. How are the radicands different? Are both students correct? Explain.
Ananya: $d = \sqrt{(5 - 3)^2 + (-2 - 1)^2}$
Jennifer: $d = \sqrt{(3 - 5)^2 + (1 - (-2))^2}$
4. Explain how to find the midpoint of a segment with endpoints at $(2, 4)$ and $(6, 2)$.

Cumulative Review

Solve.

1. $\sqrt{x} + 3 = 4$

$x = 1$

2. $3\sqrt{y} = 27$

$y = 81$

3. $\sqrt{2x - 1} = -8$

no solution

4. $3\sqrt{b} + 4 = 10$

$b = 4$

5. $\frac{2}{3}\sqrt{n} = 8$

$n = 144$

6. $\frac{4\sqrt{g}}{5} + 6 = 1$

no solution

7. $3\sqrt{x} - 2\sqrt{x} = 5$

$x = 25$

8. $\sqrt{\frac{f}{2}} + 4 = 3$

$f = 10$

Possible Journal Answers
1. The expressions are squared; so, finding the absolute value is an unnecessary step, because $|a^2| = a^2$. In other words, squaring the absolute value of an expression produces the same nonnegative result as squaring the expression.
2. Sarah incorrectly substituted the ordered pair values into the formula. She should have assigned $x_1 = 3$, $y_1 = -2$, $x_2 = 1$, and $y_2 = 4$. Substituting into the general formula the correct calculation is
$d = \sqrt{(1 - 3)^2 + (4 - (-2))^2} = \sqrt{(-2)^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}.$

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Manipulatives

Find the distance between points (–5, 1) and (3, 7) using a geoboard.

Step 1. Using bands, mark the *x*- and *y*-axes so these points will fit on the board. Place another band around the given points to form a line segment. For an 11×11 pin board, one option is an *x*-axis with minimum value –6 and maximum value 4 and *y*-axis with minimum value –1 and maximum value 9.

Step 2. Pull one side of the line segment band so a triangle is formed. Pull until one side is vertical and the other horizontal, forming a right triangle. Place the band around the point that forms this right angle. The third point of triangle should be (–5, 7) or (3, 1).

Step 3. Determine the length of the vertical and horizontal sides by counting the spaces of each.
vertical side: 6 units
horizontal side: 8 units

Step 4. Use the lengths found in Step 3 as *a* and *b* in the Pythagorean Theorem, written as $c = \sqrt{a^2 + b^2}$. Solve for *c*. This is the distance between the given points.
$c = \sqrt{6^2 + 8^2}$
$c = \sqrt{36 + 64}$
$c = \sqrt{100}$
$c = 10$ units

Find the distance between each of the given pairs of points.
Use the steps outlined above.

1. (–2, –6) and (3, 6)

third point: (–2, 6) or (3, –6)

vertical side: 12 units

horizontal side: 5 units

distance: $c = 13$ units

2. (–6, –7) and (–2, –1)

third point: (–6, –1) or (–2, –7)

vertical side: 6 units

horizontal side: 4 units

distance: $c = 2\sqrt{13}$ units

3. (2, 8) and (9, 5)

third point: (2, 5) or (9, 8)

vertical side: 3 units

horizontal side: 7 units

distance: $c = \sqrt{58}$ units

4. (–2, 1) and (7, –2)

third point: (–2, –2) or (7, 1)

vertical side: 3 units

horizontal side: 9 units

distance: $c = 3\sqrt{10}$ units

Possible Journal Answers *(continued)*
3. The difference in the radicands is that the students assigned variables differently. It does not matter which of the points is assigned as (x_1, y_1) and which as (x_2, y_2). The calculations will produce the same results since $(a - b)^2 = (b - a)^2$. Both students are correct.

4. To find the midpoint, use the formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. The *x*-coordinate of the midpoint is the average of two and six, or four. The *y*-coordinate of the midpoint is the average of four and two, or three. The midpoint is (4, 3).