NAME

Module 16 Solving Rational Equations
Lesson 1 Solving Rational Equations

independent practice

Solve the following rational equations.

1. $\frac{m}{12} = \frac{1}{3}$ **$m = 4$**
2. $\frac{p}{9} = \frac{4}{3}$ **$p = 12$**
3. $\frac{4}{x} = \frac{1}{2} + \frac{3}{x}$ **$x = 2$**
4. $\frac{3}{a} = \frac{1}{6} + \frac{2}{a}$ **$a = 6$**
5. $\frac{r}{5} = \frac{3}{5} + \frac{r}{2}$ **$r = -2$**
6. $\frac{y}{7} + \frac{3}{4} = \frac{y}{4}$ **$y = 7$**
7. $\frac{2}{5s} = \frac{1}{5}$ **$s = 2$**
8. $\frac{4}{m} = \frac{-2}{7}$ **$m = -14$**
9. $\frac{2x}{3} - \frac{5}{4} = \frac{x}{2}$ **$x = \frac{15}{2}$**
10. $\frac{3}{a} = \frac{1}{6a} - 2$ **$a = -\frac{17}{12}$**
11. $\frac{2m}{7} + \frac{3}{5} = \frac{m}{5}$ **$m = -7$**
12. $\frac{2}{3} - \frac{3b}{4} = \frac{2b}{6}$ **$b = \frac{8}{13}$**
13. $\frac{3}{2r} - \frac{5}{4} = \frac{6}{3r}$ **$r = -\frac{2}{5}$**
14. $\frac{7}{2t} + \frac{3}{t} = 4$ **$t = \frac{13}{8}$**
15. $\frac{4d}{d-2} - \frac{3}{d-2} = 5$ **$d = 7$**
16. $\frac{3g}{g+1} + \frac{2g-1}{g+1} = 3$ **$g = 2$**
17. $\frac{3}{x-4} + \frac{2x+1}{x-4} = 6$ **$x = 7$**
18. $\frac{2x}{x+3} - \frac{4x-2}{x+3} = 4$ **$x = -\frac{5}{3}$**
19. $\frac{4b}{b-6} - \frac{2b+1}{b-6} = 3$ **$b = 17$**
20. $\frac{m-6}{2m+1} + \frac{3}{2m+1} = 7$ **$m = -\frac{10}{13}$**

Journal

1. Michael says that the equation $\frac{1}{x} + \frac{3}{5} = \frac{1}{4}$ is solved by first subtracting $\frac{3}{5}$ from both sides of the equation. Describe another way to solve this equation.
2. Explain why the equation $\frac{2}{4x} + \frac{1}{3x} = \frac{8}{x}$ has no solution.
3. Sandeep claims that the only way to solve the equation $6x + \frac{3}{4} = \frac{1}{6}$ is to first multiply both sides of the equation by the LCD, 12. He is incorrect. Give two alternative methods to solve this equation.
4. Explain how to solve the equation $\frac{2}{x-6} + \frac{4x+1}{x-6} = 3$.

Possible Journal Answers

1. Another way to solve the equation $\frac{1}{x} + \frac{3}{5} = \frac{1}{4}$ is to eliminate the fractions by first multiplying each side of the equation by the least common denominator 20x. The equation is now $20 + 12x = 5x$. Collect terms $7x = -20$. This gives $x = -\frac{20}{7}$, or $-2\frac{6}{7}$.

2. When each side of the equation $\frac{2}{4x} + \frac{1}{3x} = \frac{8}{x}$ is multiplied by the least common denominator 12x, the variable x cancels in each term which gives $6 + 4 = 96$. Since $10 \neq 96$, there is no solution to this equation.

3. 1st Method:

$6x + \frac{3}{4} = \frac{1}{6}$
$6x = \frac{1}{6} - \frac{3}{4}$
$6x = \frac{2}{12} - \frac{9}{12}$
$6x = -\frac{7}{12}$
$x = \left(\frac{1}{6}\right)\left(-\frac{7}{12}\right)$
$x = -\frac{7}{72}$

2nd Method:

$6x + \frac{3}{4} = \frac{1}{6}$
$24\left(6x + \frac{3}{4}\right) = 24\left(\frac{1}{6}\right)$
$144x + 18 = 4$
$144x = 4 - 18$
$144x = -14$
$x = -\frac{14}{144}$, or $-\frac{7}{72}$

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Cumulative Review

Perform the indicated operations. Assume that the domains of the rational expressions contain no value for which any denominator is zero.

1. $\frac{3y}{y-4} - \frac{7}{y-4}$ **$\frac{3y-7}{y-4}$**

2. $\frac{3m}{m+4} \cdot \frac{4m+16}{6}$ **$2m$**

3. $\frac{2n-1}{n+3} \div \frac{n-1}{6n+18}$ **$\frac{12n-6}{n-1}$**

4. $\frac{3b+2}{b+1} + \frac{12}{b-1}$ **$\frac{3b^2+11b+10}{b^2-1}$**

5. $\frac{y^2+5y+6}{y^2-7y+12} \div \frac{2y^2+7y+3}{y^2-7y+12}$ **$\frac{y+2}{2y+1}$**

6. $\frac{2}{t^2} \cdot \left(\frac{t^3}{4t}\right)^2$ **$\frac{t^2}{8}$**

Challenge

Example: Solve.

$$\frac{3}{x-1} + \frac{4x}{3} = \frac{4}{x-1}$$ Given

$$3(x-1)\left(\frac{3}{x-1} + \frac{4x}{3}\right) = 3(x-1)\left(\frac{4}{x-1}\right)$$ Multiply by the LCD

$$9 + 4x(x-1) = 12$$ Distributive Property

$$9 + 4x^2 - 4x = 12$$ Distributive Property

$$4x^2 - 4x - 3 = 0$$ Subtraction

$$(2x+1)(2x-3) = 0$$ Factor

$$2x + 1 = 0 \text{ or } 2x - 3 = 0$$ Solve each factor for 0

$$2x = -1 \text{ or } 2x = 3$$

$$x = \frac{-1}{2} \text{ or } x = \frac{3}{2}$$

Checking the answer of $-\frac{1}{2}$ gives

$$\frac{3}{x-1} + \frac{4x}{3} = \frac{4}{x-1}$$

$$\frac{3}{\left(\frac{-1}{2}\right)-1} + \frac{4 \cdot \left(\frac{-1}{2}\right)}{3} \stackrel{?}{=} \frac{4}{\left(\frac{-1}{2}\right)-1}$$

$$3\left(-\frac{2}{3}\right) + \left(-\frac{2}{3}\right) \stackrel{?}{=} 4\left(-\frac{2}{3}\right)$$

$$-\frac{8}{3} = -\frac{8}{3}$$

Checking the answer of $\frac{3}{2}$ gives

$$\frac{3}{x-1} + \frac{4x}{3} = \frac{4}{x-1}$$

$$\frac{3}{\left(\frac{3}{2}\right)-1} + \frac{4 \cdot \left(\frac{3}{2}\right)}{3} \stackrel{?}{=} \frac{4}{\left(\frac{3}{2}\right)-1}$$

$$3(2) + \frac{6}{3} \stackrel{?}{=} 4(2)$$

$$8 = 8$$

The solution set to this equation is $x = -\frac{1}{2}$ and $x = \frac{3}{2}$.

Solve.

1. $\frac{3}{x} = \frac{x}{12}$ **$x = 6$ or -6**

2. $\frac{3}{x} - 4 = \frac{4x}{3-x}$ **$x = \frac{3}{5}$**

3. $\frac{3}{4m} + \frac{2m}{m-2} = 2$ **$m = \frac{6}{19}$**

4. $\frac{1}{6y} + \frac{3y}{2y+4} = \frac{4}{2y+4}$ **$y = \frac{2}{9}$ or 1**

5. $\frac{s-3}{s} + \frac{s-4}{s-6} = \frac{1}{s}$ **$s = 3$ or 4**

Possible Journal Answers *(continued)*

4. Begin by multiplying each side of the equation by $x - 6$. This yields the equation $2 + 4x + 1 = 3x - 18$. Combine like terms to get $3 + 4x = 3x - 18$. Subtract three from both sides of the equation to get $4x = 3x - 21$. Subtract $3x$ from both sides of the equation. The solution is $x = -21$.