NAME

Module 13 Solving Quadratic Equations of One Variable

Lesson 6 Solving Problems Using Quadratic Equations of One Variable

independent practice

Solve.

1. The area of a rectangular porch is 105 square feet. The width of the porch is w feet, and the length is $3w - 6$ feet. What are the dimensions of the porch?

   $105 = (3w - 6)w$; $w = 7$; The porch dimensions are 7 feet by 15 feet.

2. The length of a rectangular patio is twice the width. The area of the patio is 800 square feet. What are the dimensions of the patio?

   $800 = 2w(w)$; $w = 20$; The patio dimensions are 20 feet by 40 feet.

3. The area of a square message board is 256 square inches. What is the length of a side of the message board?

   $256 = s^2$; $s = 16$; The message board side measures 16 inches.

4. The area of the floor in Samantha's rectangular room is 120 square feet. The length of the room is eight feet less than twice the width. What are the dimensions of the floor?

   $120 = (2w - 8)w$; $w = 10$; The floor dimensions are 10 feet by 12 feet.

5. If the sides of a square are increased by three centimeters, its area becomes 81 square centimeters. What was the length of the side of the original square?

   $(s + 3)^2 = 81$; $s = 6$; The original square has sides which measure 6 centimeters each.

6. The height, in feet, of a softball thrown upwards from a point one hundred feet above the ground is given by the equation $h = -16t^2 + 5t + 100$, where t is time in seconds. How many seconds will have elapsed when the softball is 25 feet above the ground?

   $25 = -16t^2 + 5t + 100$; $t \approx 2.33$; The softball is 25 feet above the ground after about 2.33 seconds.

7. The height, in feet, of a football kicked into the air is given by the equation $h = -16t^2 + 75t + 2$, where t is time in seconds. If the football is kicked, how much time will the football be in the air?

$0 = -16t^2 + 75t + 2$; $t \approx 4.71$; The football is in the air for about 4.71 seconds.

8. The height, in feet, of a football kicked into the air is given by the equation $h = -16t^2 + 75t + 2$, where t is time in seconds. How many seconds will have elapsed when the ball is 90 feet above the ground?

The discriminant of $0 = -16t^2 + 75t - 88$ is –7. There are no real solutions to this equation; so, the ball does not reach a height of 90 feet.

9. A good swimmer runs and jumps from a cliff 10 meters above the water. Her height in meters above the water t seconds after she jumps is given by $h = -4.9t^2 + 4t + 10$. How many seconds is she in the air before she reaches the water?

$0 = -4.9t^2 + 4t + 10$; $t \approx 1.89$; She was in the air about 1.89 seconds.

10. In a drop tower experiment to test durability and shock absorption, a backpack prototype is dropped from a height of 50 feet. The equation $h = -16t^2 + 50$ gives the height h of the backpack in feet, t seconds after it is dropped. Find the height of the backpack two seconds after it is dropped.

$h = -16(2^2) + 50 = -14$; After two seconds the height is negative. So, the backpack already hit the ground. Its height after two seconds is zero.

Solve the following problems by using the techniques learned in the lesson for solving word problems using quadratic equation of one variable. The scenarios, however, differ from those presented in the lesson.

11. The product of two numbers is 180. One number is five times the other number. What are the numbers?

$n(5n) = 180$; The numbers are either 6 and 30 or –6 and –30.

12. The sum of a number and the square of the number is 20. What is the number and its square?

$n + n^2 = 20$; The numbers are either 4 and 16 or –5 and 25.

13. The revenue, R, in dollars of a tour bus is given by $R = -x^2 + 10x + 3{,}000$, where x is the number of unsold seats. How much revenue will the tour bus company receive when all seats are sold?

$R = -0^2 + 10(0) + 3{,}000 = 3{,}000$; So, the company receives \$3,000 in revenue when all seats are sold.

14. The revenue, R, in dollars of a tour bus is given by $R = -x^2 + 10x + 3{,}000$, where x is the number of unsold seats. How many unsold seats will result in \$0 revenue?

$0 = -x^2 + 10x + 3{,}000$. So, the company makes \$0 when 60 seats are unsold.

15. A political fundraiser usually sells 50 banquet tables when the price is \$2k per table, but for every \$1k increase in table price, the number of tables sold decreases by five. How much revenue will be raised when the table price is \$4k? Hint: revenue = (price per table) × (number of tables sold). In the financial world, "k" is shorthand for "thousand", so \$2k = \$2,000, but it is easier to work this problem without converting the k's.

R = (price)(tables sold); x is the \$1k price increase; $R = (2 + x)(50 - 5x)$; $R = -5x^2 + 40x + 100$; $R = -5(2^2) + 40(2) + 100 = 160$; So, the fundraiser will raise \$160k or \$160,000 when the price of the table is \$4k.

16. Use the fundraiser information of Problem 15 to find the price of a table if the desired revenue is \$175k. Also find the number of tables sold at this price.

$175 = -5x^2 + 40x + 100$; $0 = 5x^2 + 40x - 75$; $x = 3$ or $x = 5$; The fundraiser will raise \$175k when either 35 tables are sold at \$5k or when 25 tables are sold at \$7k.

Journal

1. Rachel's mother is making a rectangular quilt with a length that is two feet less than twice the width. The area of the quilt is 40 square feet. Explain the process for determining the dimensions of the quilt.
2. The height, in feet, of an object thrown in the air is given by the equation $h = -16t^2 + 9t + 25$, where t is the time in seconds. What is the significance of the coefficients –16 and nine and the constant term 25?
3. Leah wants wall-to-wall carpeting. Her room is rectangular, and one wall is five feet shorter than two and a half times the length of the adjacent wall. The area of her room is 120 square feet. Would a carpet that is 14 feet long by eight feet wide be large enough to cover her bedroom floor? Explain.
4. If a ball is thrown into the air with an initial velocity of 16 feet per second from an initial height of 45 feet from the ground, how long is it before it reaches a height of 300 feet? Explain your reasoning.
5. In solving real world problems which are modeled by a quadratic equation of one variable, the typical solution set contains two numbers. Do you always reject a negative solution? If possible give three real world scenarios in which a negative solution would be acceptable.

Cumulative Review

Determine if the equation is quadratic, linear or neither.

1. $(a + 2)(a - 2) = 0$

Quadratic

2. $d^3(d^2 + 5d) = d^5 + d^2$

Neither

3. $r^2(r + 5) = r^3 + 5r^2 + 9r$

Linear

4. $6z^2 - 2z = 5z^2 - 4z + 1$

Quadratic

Solve.

5. $3x^2 - 12 = 63$

$x = 5$ **or** $x = -5$

6. $3(x + 3)^2 + 21 = 96$

$x = 2$ **or** $x = -8$

7. $4(x + 3)^2 + 28 = 428$

$x = 7$ **or** $x = -13$

8. $(x + 8)(x - 3) = 0$

$x = 3$ **or** $x = -8$

9. $9x^2 - 12x - 7 = 5$

$x = 2$ **or** $x = -\frac{2}{3}$

10. $6x^2 + 19x = 4x$

$x = 0$ **or** $x = -2.5$

Complete the square.

11. $x^2 - 14x +$ **49**

12. $x^2 + 18x +$ **81**

Possible Journal Answers

1. Area is equal to length times width. To solve, the area formula needs to be written in terms of one variable. Since the length is two feet less than twice the width, $l = 2w - 2$. Substituting this expression for l in the formula $A = lw$ results in $A = (2w - 2)w$. Since the total area is 40 square feet, $40 = (2w - 2)w$, or $40 = 2w^2 - 2w$. To solve for w, subtract 40 from both sides and then, divide all terms in the equation by two. The resulting equation is: $0 = w^2 - w - 20$. This can be rewritten as a product of two binomials: $(w - 5)(w + 4) = 0$. So, $w = 5$ or $w = -4$. Because width is a measurement, it cannot have a negative value. Therefore, w must be equal to five. Since $l = 2w - 2$, substitute five for w and solve. The length is eight and the width is five.
2. The coefficient nine is the initial velocity of the object when it is thrown. The constant term, 25, is the value of the initial height.
3. No. The carpet must be at least 15 feet long and eight feet wide. Since the area of her carpet is 120 square feet and the length of the room is five feet less than 2.5 times the width of the room, $120 = w(2.5w - 5)$. Solving for w yields eight feet. Substituting back into the equation $l = 2.5w - 5$ yields a length of 15. Some students may simply multiple 14 by eight to get 112 square feet, which is less than the required 120 square feet.
4. The ball will never reach 300 feet. To find the time, t, solve the equation $300 = -16t^2 + 16t + 45$. To solve for t, subtract 300 from both sides to arrive at $0 = -16t^2 + 16t - 255$. Then, use the quadratic formula to solve for t: $t = \frac{-16 \pm \sqrt{16^2 - 4(-16)(-255)}}{2(-16)}$ or $t = \frac{-16 \pm \sqrt{-16{,}064}}{-32}$. Since the radicand is negative, there is no solution to this problem.
5. Depending on the real world situation, negative solutions are usually rejected as not being feasible. Dimensions, time, weights, measures, and amounts of physical quantities are all examples of quantities for which negative values are usually not reasonable and must be rejected as solutions. However, there are real world situations for which negative-valued solutions are possible. Some examples include stock market changes, temperatures, diet results, and business profits (losses).