NAME ___

Module 13 Solving Quadratic Equations of One Variable

Lesson 4 Solving Quadratic Equations by Completing the Square

independent practice

Complete the square.

1. $x^2 + 14x +$ **49**

2. $x^2 - 12x +$ **36**

3. $y^2 + 3y +$ $\frac{9}{4}$

4. $n^2 - 5n +$ $\frac{25}{4}$

5. $H^2 +$ **16** $H + 64$

6. $x^2 -$ **18** $x + 81$

Factor.

7. $x^2 + 6x + 9$

$(x + 3)^2$

8. $z^2 + 12z + 36$

$(z + 6)^2$

9. $m^2 - 7m + \frac{49}{4}$

$\left(m - \frac{7}{2}\right)^2$

10. $x^2 - 11x + \frac{121}{4}$

$\left(x - \frac{11}{2}\right)^2$

Solve by completing the square.

11. $x^2 + 10x = 1$

$\{-5 + \sqrt{26}, -5 - \sqrt{26}\}$

12. $x^2 + 14x = -2$

$\{-7 + \sqrt{47}, -7 - \sqrt{47}\}$

13. $c^2 - 8c - 5 = 0$

$\{4 + \sqrt{21}, 4 - \sqrt{21}\}$

14. $P^2 - 18P + 28 = 0$

$\{9 + \sqrt{53}, 9 - \sqrt{53}\}$

15. $x^2 + 2 = 11 - 8x$

$\{-9, 1\}$

16. $x^2 + 14 = 6x + 5$

$\{3\}$

17. $3r^2 - 12r + 4 = 10$

$\{2 + \sqrt{6}, 2 - \sqrt{6}\}$

18. $4H^2 + 32H + 20 = 4$

$\{-4 + 2\sqrt{3}, -4 - 2\sqrt{3}\}$

19. $20x + 5x^2 + 30 = 10$

$\{-2\}$

20. $9 + 2B^2 - 6B = 5$

$\{1, 2\}$

21. $8 + 3w^2 + 6w = 5$

$\{-1\}$

22. $3x + 9x^2 - 7 = -3$

$\left\{\frac{-1 - \sqrt{17}}{6}, \frac{-1 + \sqrt{17}}{6}\right\}$

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Journal

1. Abe and Sarah were given the expression $x^2 - 16x +$ ____ and asked to complete the square. Abe said the answer is $x^2 - 16x + 256$, and Sarah said the answer is $x^2 - 16x + 8$. Is either student correct? Explain.
2. What are the steps for solving a quadratic equation by completing the square?
3. Explain the process for completing the square in the expression $x^2 -$ ____$x + 36$.
4. Richard and Janelle are challenged with solving the equation $x^2 - 12x + 17 = 13$. Richard believes the solutions are 13 and -1. Janelle believes the solutions are $6 + \sqrt{32}$ and $6 - \sqrt{32}$. Is either student correct? Describe any possible errors and explain the process for finding the correct solutions.
5. True or False: The solution set for the equation $7x^2 + 4x + 3 = 5$ is $\left\{-\frac{2}{7} + \frac{\sqrt{18}}{7}, -\frac{2}{7} - \frac{\sqrt{18}}{7}\right\}$. Explain.

Cumulative Review

Determine if the equation is quadratic, linear or neither.

1. $2x^2 - 3x = 2x^2 - 4x + 1$

   Linear

2. $(z + 2)(z - 2) = 0$

   Quadratic

3. $d^2(d + 5) = d^3 + 9$

   Quadratic

4. $f^2(f^2 + 5f) = f^4 + f^2$

   Neither

Solve by factoring or by evaluating square roots.

5. $4x^2 - 11 = 53$

   {–4, 4}

6. $3(x + 4)^2 - 16 = 32$

   {–8, 0}

7. $(x - 5)(x + 3) = -7$

   {4, –2}

8. $(x + 7)(x - 4) = 0$

   {4, –7}

9. $4x^2 + 12x - 7 = 9$

   {–4, 1}

10. $-5x^2 + 17x = 2x$

   {0, 3}

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Graphing Calculator Problem

Follow the steps below to solve the equation $x^2 + 8x - 20 = 10$ by graphing the associated quadratic function and finding the *x*-intercepts. When $y = 0$ at the *x*-intercepts, the associated quadratic function becomes the original equation.

1. First, replace the equation with an equivalent equation of the form $ax^2 + bx + c = 0$. In this case, subtract 10 from both sides of the equation. The equation becomes $x^2 + 8x - 30 = 0$.
2. Enter the associated quadratic function $y = x^2 + 8x - 30$ into the calculator: Press [Y=] then [CLEAR] (if needed). With the cursor on the line Y_1= (use the arrow keys to move it there, if necessary), press [X,T,θ,n] [x^2] [+] [8] [X,T,θ,n] [−] (do NOT use the [(−)] key) [3] [0]. See Figure 1.
3. Graph the function; press [ZOOM] [6] to use the standard window. See Figure 2. This window does not show enough of the graph; press [ZOOM] [3] [ENTER] to zoom out.
4. The *x*-intercepts (the *x*-values of the points where the graph crosses the *x*-axis) are the *x* values that make $y = 0$. This means that the *x*-intercepts make $x^2 + 8x - 30 = 0$ true and are, therefore, the solutions to the original equation.
5. Find the (approximate) first *x*-intercept; press [2nd] [CALC] [2]. **Left Bound?** will appear in the lower left hand corner of the screen. Use the left and right arrow keys to move the cursor just above the x-axis, to the left of what appears to be the first *x*-intercept. Press [ENTER]. **Right Bound?** will appear in the lower left hand corner of the screen. Use the right arrow key to move the cursor just below the *x*-axis to the right of this *x*-intercept. Press [ENTER]. **Guess?** will appear in the lower left hand corner of the screen; press [ENTER]. **x = –10.78233 y = 0** appear in the lower left hand corner of the screen. See Figure 3. These values represent the point (–10.78233, 0) on the graph, where –10.78233 is the calculator's decimal approximation for the true solution, which is irrational. Write this approximate solution on your paper.
6. Repeat Step 5 to identify the approximate value of the other *x*-intercept. Remember to use the left and right arrow keys to move the cursor just to the left of the second *x*-intercept and then just to the right of the second *x*-intercept. See Figure 4. You should get 2.78233 as the calculator's decimal approximation.
7. Finally, write the set of approximate solutions for the equation $x^2 + 8x - 20 = 10$: {–10.78233, 2.78233}

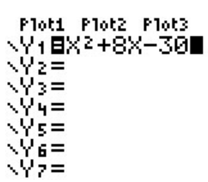

Figure 1

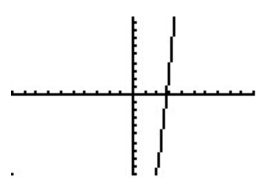

Figure 2

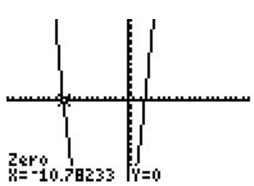

Figure 3

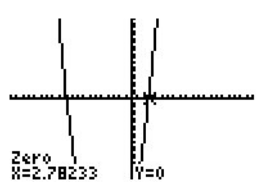

Figure 4

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Use the graphing calculator to solve (find the approximate roots of) these equations.

1. $4x^2 - 9x + 4 = 6$

{–0.2037682, 2.4537682}

2. $-3n^2 + 9n - 1 = 2$

{0.38196601, 2.618034}

3. $-17D^2 - 31D + 7 = -12$

{–2.307817, 0.484228756}

4. $\frac{1}{6}x^2 - 8x + 7 = 27$

{–2.381812, 50.381812}

Possible Journal Answers

1. **Neither student is correct. Abe forgot to divide the coefficient of the *x*-variable by two before squaring. Sarah remembered to divide the coefficient of the *x*-variable by two but forgot to square the result. The correct approach is to divide –16 by two and then, square the result: $(-16 \div 2)^2 = (-8)^2 = 64$. The completed square is $x^2 - 16x + 64$.**
2. **First, isolate variable terms on one side of the equation. Next, make the coefficient of x^2 equal to one. If the coefficient is something other than one, then divide every term in the equation by that coefficient. Third, complete the square. To do this, take half of the coefficient of the *x*-term and square it. Add the result to both sides of the equation. After completing the square, factor the trinomial and then, solve for *x*.**
3. **Find the square root of the third term, 36, and then, double that square root. The square root of 36 is six, and 2(6) = 12. So, after completing the square the expression is $x^2 - 12x + 36 = 0$.**
4. **Janelle is correct. Richard probably replaced 17 on the left side of the equation with 36 and added 36 to the right side of the equation. This is incorrect because by replacing 17 with 36 he, in effect, added 19 to the left side. Janelle correctly subtracted 17 from both sides of the equation and then completed the square by adding 36 to both sides. At that point, the equation was $x^2 - 12x + 36 = 32$. Then, she expressed the left side as a binomial square, getting $(x - 6)^2 = 32$. Next, Janelle reasoned that if $(x - 6)^2 = 32$, then $x - 6 = \sqrt{32}$ or $x - 6 = -\sqrt{32}$. Finally, she isolated the variable *x* on the left side of each equation by adding six to both sides of each equation.**
5. **True. Isolate the *x* terms by subtracting three from both sides of the equation. Divide every term in the equation by seven to make the coefficient of the x^2 term one. The coefficient of the middle term becomes $\frac{4}{7}$. Half of $\frac{4}{7}$ is $\frac{2}{7}$. Square $\frac{2}{7}$ and then, add the result to both sides of the equation to get $x^2 + \frac{4}{7}x + \frac{4}{49} = \frac{2}{7} + \frac{4}{49}$. Factor the left side and add terms on the right side to arrive at $\left(x + \frac{2}{7}\right)^2 = \frac{18}{49}$. Using square roots, $x + \frac{2}{7} = \frac{\sqrt{18}}{7}$ or $x + \frac{2}{7} = -\frac{\sqrt{18}}{7}$ Finally, subtract $\frac{2}{7}$ from both sides of each equation.**

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