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Module 12 Factoring Using Several Methods
Lesson 6 Dividing Polynomials by Monomials

independent practice

Factor completely.

1. $3x^3 - 48x$

   $3x(x - 4)(x + 4)$

2. $y^4 - 81y^2$

   $y^2(y - 9)(y + 9)$

3. $12x^3 - 108x$

   $12x(x + 3)(x - 3)$

4. $-4c^3 + 196c$

   $-4c(c - 7)(c + 7)$

5. $3d^3 + 21d^2 + 36d$

   $3d(d + 4)(d + 3)$

6. $2x^3 + 6x^2 - 20x$

   $2x(x + 5)(x - 2)$

7. $5a^3 - 40a^2 + 75a$

   $5a(a - 3)(a - 5)$

8. $3p^2q + 12pq - 63q$

   $3q(p + 7)(p - 3)$

9. $8z^2 + 28z + 12$

   $2(2z + 1)(2z + 6)$

10. $12f^3 - 2f^2 - 4f$

    $2f(3f - 2)(2f + 1)$

11. $6d^3 + 2d^2 - 8d$

    $2d(3d + 4)(d - 1)$

12. $12m^3n + 2m^2n - 80mn$

    $2mn(3m + 8)(2m - 5)$

13. $r^3 + 2r^2 - 16r - 32$

    $(r + 4)(r - 4)(r + 2)$

14. $2b^5 - 32b$

    $2b(b^2 + 4)(b + 2)(b - 2)$

15. $6m^6 - 12m^4 - 48m^2$

    $6m^2(m^2 + 2)(m + 2)(m - 2)$

16. $162n^9 - 288n^7 + 288n^5 - 512n^3$

    $2n^3(9n^4 + 16)(3n + 4)(3n - 4)$

17. $a^2b + 3a^2 - 36b - 108$

    $(a + 6)(a - 6)(b + 3)$

18. $3c^2d^2 + 21c^2d - 48d^2 - 336d$

    $3d(c + 4)(c - 4)(d + 7)$

19. $-2f^2g^2 + 10f^2g + 18g^2 - 90g$

    $-2g(f + 3)(f - 3)(g - 5)$

20. $2x^3y^2 - 18x^3 + 32xy^2 - 288x$

    $2x(x^2 + 16)(y + 3)(y - 3)$

Journal

1. Raoul believes that the simplest factored form of $x^4 - 16$ is $(x^2 + 4)(x^2 - 4)$. Explain why he is incorrect and provide the correct answer.
2. Describe the process for factoring $z^3 + 5z^2 - z - 5$.
3. Explain the steps for completely factoring $16m^4 - 81n^4$.
4. Ramzi and Sashi have been discussing the difference of two squares. Ramzi states that the completely factored form of $-3a^3 - 3ab^2$ is $-3a(a^2 - b^2)$, but Sashi insists that the completely factored form is $-3a(a + b)(a - b)$. Is either student correct? Explain why or why not.

Cumulative Review

Simplify.

1. $14x^2 + 28$

   $14(x^2 + 2)$

2. $-2m^3 - 16m$

   $-2m(m^2 + 8)$

3. $-(a + b) + c(a + b)$

   $(a + b)(c - 1)$

4. $cd + 5 + 5d + c$

   $(c + 5)(d + 1)$

5. $81 - 4z^2$

   $(9 + 2z)(9 - 2z)$

6. $p^4 - 81$

   $(p^2 + 9)(p + 3)(p - 3)$

7. $x^2 - 2x - 63$

   $(x + 7)(x - 9)$

8. $g^2 - 16g + 39$

   $(g - 3)(g - 13)$

9. $5q^2 - 29q - 6$

   $(5q + 1)(q - 6)$

10. $-6n^3 - 10n^2 + 56n$

    $2n(-3n + 7)(n + 4)$ or $-2n(3n - 7)(n + 4)$

Possible Journal Answers

1. For an expression to be considered factored completely, all the factors must be monomials or prime polynomials. Raoul's solution contains one prime polynomial, $(x^2 + 4)$, and one polynomial that can be factored further, $(x^2 - 4)$. The polynomial $(x^2 - 4)$ can be factored into $(x + 2)(x - 2)$. So, factored completely, the answer is $(x^2 + 4)(x + 2)(x - 2)$.

2. This expression can be factored by grouping. Begin by rewriting it as $(z^3 - z) + (5z^2 - 5)$. Because z^3 and z have a common factor, z, $(z^3 - z)$ can be factored as $z(z^2 - 1)$. Because $5z^2$ and 5 have a common factor, 5, $(5z^2 - 5)$ can be factored as $5(z^2 - 1)$. The expression becomes $z(z^2 - 1) + 5(z^2 - 1)$. Because $(z^2 - 1)$ is a factor common to both terms, $z(z^2 - 1) + 5(z^2 - 1)$ can be rewritten as $(z^2 - 1)(z + 5)$. Because $(z^2 - 1)$ is a difference of two squares, it can be factored as $(z - 1)(z + 1)$. The complete factorization is $(z + 5)(z - 1)(z + 1)$.

3. This is the difference of two squares because $16m^4$ can be written as $(4m^2)^2$ and $81n^4$ can be written as $(9n^2)^2$. Therefore, $16m^4 - 81n^4$ can be written as $(4m^2)^2 - (9n^2)^2$ and factored as $(4m^2 + 9n^2)(4m^2 - 9n^2)$. Because $(4m^2 - 9n^2)$ is also a difference of two squares, it can be factored as $(2m + 3n)(2m - 3n)$. Therefore, the fully factored expression is $(4m^2 + 9n^2)(2m + 3n)(2m - 3n)$.

4. Neither Ramzi nor Sashi is correct. Ramzi made an error when he factored out the $-3a$. When $-3a$ is factored out of both terms in the binomial, the expression can be rewritten as $-3a(a^2 + b^2)$. This cannot be factored further. Sashi made the same mistake but took it one step further. He mistakenly thought that the partially factored expression was $-3a(a^2 - b^2)$ and then, factored the difference of two squares. But there is no difference of two squares because, after factoring out $-3a$, the expression is $-3a(a^2 + b^2)$.