NAME ______________________________

Module 12 Simplifying Algebraic Expressions by Factoring Polynomials

Lesson 5 Factoring $ax^2 + bx + c$

independent practice

Factor.

1. $2x^2 + 9x + 7$

$(2x + 7)(x + 1)$

2. $3x^2 + 8x + 5$

$(3x + 5)(x + 1)$

3. $5x^2 + 11x + 2$

$(5x + 1)(x + 2)$

4. $7x^2 + 2x + 5$

Cannot be factored; Prime

5. $4x^2 + 13x + 3$

$(4x + 1)(x + 3)$

6. $7x^2 - 4x - 3$

$(7x + 3)(x - 1)$

7. $11x^2 - 6x - 5$

$(11x + 5)(x - 1)$

8. $5x^2 - 14x - 3$

$(5x + 1)(x - 3)$

9. $13x^2 - 2x - 15$

$(13x - 15)(x + 1)$

10. $5x^2 + 2x - 7$

$(5x + 7)(x - 1)$

11. $17x^2 + 33x - 2$

$(17x - 1)(x + 2)$

12. $3x^2 - 10x + 3$

$(3x - 1)(x - 3)$

13. $5x^2 - 12x + 7$

$(5x - 7)(x - 1)$

14. $2x^2 + x - 3$

$(x - 1)(2x + 3)$

15. $8x^2 + 2x - 15$

$(4x - 5)(2x + 3)$

16. $5x^2 + 7x - 24$

$(5x - 8)(x + 3)$

17. $9x^2 + 24x + 16$

$(3x + 4)^2$ or $(3x + 4)(3x + 4)$

18. $2x^2 - x - 3$

$(2x - 3)(x + 1)$

19. $12x^2 - 23x + 5$

$(3x - 5)(4x - 1)$

20. $7x^2 - 16x + 9$

$(7x - 9)(x - 1)$

Journal

1. Aaron insists that the factored form of $4x^2 - 12x + 5$ is $(2x + 1)(2x + 5)$. Explain what Aaron did correctly, but why his factorization is incorrect. What would the trinomial need to be for his factorization to be correct?
2. Create a trinomial of the form $ax^2 + bx + c$, where $b > 0$ and $c > 0$, and a and c are prime. Explain each step for factoring it.
3. Bruce thinks the only way to factor $6x^2 + 11x + 4$ is to use the traditional method of finding the factor pairs of the first term, to separate the pairs into two binomials, and then, to use *guess-and-check* with factor pairs of the third term to see what works. Explain to Bruce another way to factor this trinomial.
4. Can a trinomial whose first term is negative be factored into a product of two binomials? Explain your answer using an example.

Cumulative Review

Simplify.

1. $8f + 20$

   $4(2f + 5)$

2. $15m^2 - 15m - 40$

   $5(3m^2 - 3m - 8)$

3. $7s^2t + 3s - 10t$

   Cannot be factored; prime

4. $18a^3b^4 + 9a^2b^3 - 12a^2b^2$

   $3a^2b^2(6ab^2 + 3b - 4)$

5. $gh - 4g + 2h - 8$

   $(h - 4)(g + 2)$

6. $xz + 6x - yz - 6y$

   $(x - y)(z + 6)$

7. $16r^2 - 12r - 12r + 9$

   $(4r - 3)(4r - 3)$ or $(4r - 3)^2$

8. $9m^2 - 16n^2$

   $(3m + 4n)(3m - 4n)$

9. $x^2 + 21x + 38$

   $(x + 19)(x + 2)$

10. $x^2 + 15x - 54$

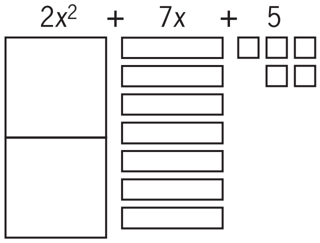
    $(x + 18)(x - 3)$

Manipulatives

Use algebra tiles to factor. $2x^2 + 7x + 5$ with tiles. Begin by modeling the trinomial.

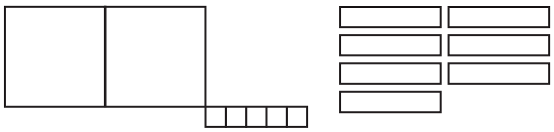
Figure 1

$2x^2 \quad + \quad 7x \quad + \quad 5$



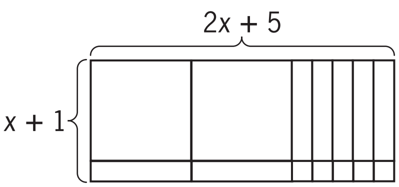
Put the x^2-squares in a row and then arrange the 1's tiles, so they form a rectangle. Because five is a prime number, the only rectangle that can be formed is a 1×5 rectangle. Now arrange the tiles so the lower, right corner of the x^2's rectangle and the upper left corner of the 1's rectangle are touching.

Figure 2



Finally, fill in the *x*-rectangles above and to the left of the 1-squares to form a rectangle. All tiles should be used in forming a rectangle. If there are too few *x*-rectangles or if there are *x*-rectangles left over, try adding zero pairs, or start over with a different configuration of 1's tiles.

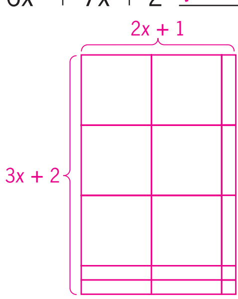
Figure 3



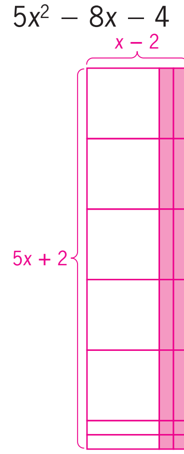
$2x^2 + 7x + 5 = (2x + 5)(x + 1)$

Use algebra tiles to simplify the following:

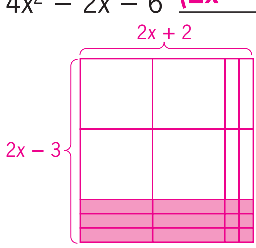
1. $6x^2 + 7x + 2$ **(3x + 2)(2x + 1)**



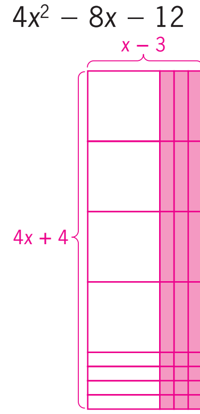
2. $5x^2 - 8x - 4$ **(5x + 2)(x − 2)**



3. $4x^2 - 2x - 6$ **(2x − 3)(2x + 2)**



4. $4x^2 - 8x - 12$ **(4x + 4)(x − 3)**



Possible Journal Answers

1. Aaron placed the factors of the first term in the correct location. He also recognized that the last term was positive. His error was when he used the wrong signs for the factors of the last term. Because the last term of the trinomial is positive, its factors must both be positive or both be negative. If they are both positive, then the coefficient of the middle term would also be positive, but the coefficient of the middle term of the trinomial is negative. Therefore, Aaron should have used the negative factors of last term. His solution should have been $(2x - 1)(2x - 5)$. The trinomial would have had to be $4x^2 + 12x + 5$ for Aaron's factorization to be correct.
2. There are many correct answers. One example is given here. Example: $3x^2 + 7x + 2$. First, factor the first term of the trinomial, $3x^2$, and list the factors in parentheses: $(3x \quad)(x \quad)$. Next, find the factors of the last term in the trinomial. Because the last term is prime, it has only two factors, one and two. List all ways the factors of the third term can be arranged in the two binomials. This example has no negative numbers, so there are only two possibilities: $(3x + 2)(x + 1)$ or $(3x + 1)(x + 2)$. Multiply each pair to see which matches original trinomial. Because $(3x + 2)(x + 1) = 3x^2 + 5x + 2$ and $(3x + 1)(x + 2) = 3x^2 + 7x + 2$, the answer is $(3x + 1)(x + 2)$.
3. This can be factored using the following steps: 1) Multiply the coefficient of the first term and the constant term. The coefficient of the first term is six, and the constant term is four. So, $6 \times 4 = 24$. 2) Because both the coefficient of the second term and the constant term are positive, find all pairs of positive factors with a product of 24. They are (1×24), (2×12), (3×8), and (4×6). 3) Determine which factor pair adds up to the coefficient of the middle term of the trinomial 11. In this case, the sum of eight and three is 11. 4) Rewrite the trinomial replacing the coefficient of the second term with the sum of the factors of the first term: $6x^2 + 11x + 4 = 6x^2 + 3x + 8x + 4$. 5) Recognize that the four terms in this new expression can be grouped into two binomials. Then, factor out common terms in each binomial: $6x^2 + 3x + 8x + 4 = (6x^2 + 3x) + (8x + 4) = 3x(2x + 1) + 4(2x + 1)$. 6) Because $(2x + 1)$ is multiplied by $3x$ and by 4, the expression can be rewritten as $(3x + 4)(2x + 1)$. This is the factored form.
4. The coefficient of the first term of a trinomial can be positive or negative. For example, $-5x^2 + 12x - 7$. The first term multiplied by the constant term is $(-7) \cdot (-5) = 35$. List the pairs of factors that multiply to equal 35. They are (1×35), (5×7) and (-1×-35), (-5×-7). The sum of five and seven is 12, so this is the factor pair that will work. Substituting in $5 + 7$ for 12, the new trinomial is $-5x^2 + 5x + 7x - 7$. This can be rewritten as $(-5x^2 + 5x) + (7x - 7)$. Factoring negative five out of the first binomial gives $-5x(x - 1)$, and factoring seven out of the second binomial gives $7(x - 1)$. The result is $-5x(x - 1) + 7(x - 1)$ or $(-5x + 7)(x - 1)$.