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Module 12 Simplifying Algebraic Expressions by Factoring Polynomials

Lesson 4 Factoring $x^2 + bx + c$

independent practice

Factor, if possible.

1. $x^2 + 4x + 3$

$(x + 3)(x + 1)$

2. $u^2 + 9u + 18$

$(u + 3)(u + 6)$

3. $a^2 + 11a + 28$

$(a + 7)(a + 4)$

4. $b^2 + 11b + 24$

$(b + 8)(b + 3)$

5. $p^2 + 7p + 14$

Cannot be factored; prime

6. $m^2 + 9m + 20$

$(m + 5)(m + 4)$

7. $k^2 - 5k + 6$

$(k - 3)(k - 2)$

8. $d^2 - 7d + 12$

$(d - 3)(d - 4)$

9. $n^2 - 9n + 14$

$(n - 7)(n - 2)$

10. $r^2 - 8r + 16$

$(r - 4)^2$ or $(r - 4)(r - 4)$

11. $y^2 - 13y + 36$

$(y - 9)(y - 4)$

12. $z^2 - 10z + 24$

$(z - 6)(z - 4)$

13. $h^2 - 2h - 8$

$(h - 4)(h + 2)$

14. $v^2 - v - 12$

$(v - 4)(v + 3)$

15. $w^2 - 4w - 3$

Cannot be factored; prime

16. $q^2 - 2q - 48$

$(q - 8)(q + 6)$

17. $a^2 - 11a - 42$

$(a - 14)(a + 3)$

18. $m^2 - 12m - 64$

$(m - 16)(m + 4)$

19. $f^2 + 4f - 5$

$(f + 5)(f - 1)$

20. $c^2 + 7c - 18$

$(c + 9)(c - 2)$

21. $n^2 + 5n - 14$

$(n + 7)(n - 2)$

22. $t^2 + 5t - 24$

$(t + 8)(t - 3)$

23. $g^2 + 8g - 20$

$(g + 10)(g - 2)$

24. $s^2 + s - 42$

$(s + 7)(s - 6)$

Journal

1. Explain why the trinomial $z^2 - 7z - 10$ cannot be simplified into two binomial factors.
2. Nicholas says the factored form of $x^2 - 3x - 18$ is $(x + 6)(x - 3)$. Explain why his solution is incorrect. What would the trinomial need to be for his solution to be correct?
3. If both the second and third terms in a trinomial are negative, what must be true about its binomial factors? Explain.
4. Create a trinomial of the form $x^2 + bx + c$, where $b > 0$ and $c > 0$, which can be factored. Explain each step for factoring it.
5. Explain how factoring a trinomial is related to the FOIL Method.

Cumulative Review

Factor, if possible.

1. $3b + 9$

$3(b + 3)$

2. $12z^2 - 18z - 6$

$6(2z^2 - 3z - 1)$

3. $9c^2d + 3cd^2 - 15c$

$3c(3cd + d^2 - 5)$

4. $p(m + n) + 2(m + n)$

$(m + n)(p + 2)$

5. $4r^2 - 2rq - 2rq + q^2$

$(2r - q)(2r - q)$ or $(2r - q)^2$

6. $2s^2 + 3st - 2st - 3t^2$

$(s - t)(2s + 3t)$

7. $49x^2 - 16$

$(7x - 4)(7x + 4)$

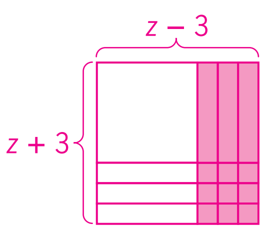
8. $25n^2 - 4$

$(5n - 2)(5n + 2)$

Factor using algebra tiles.

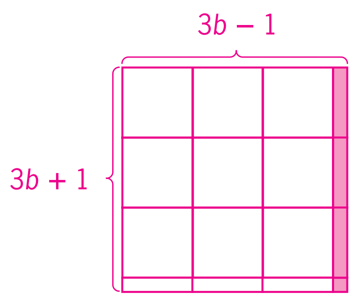
9. $z^2 - 9$

$(z - 3)(z + 3)$



10. $9b^2 - 1$

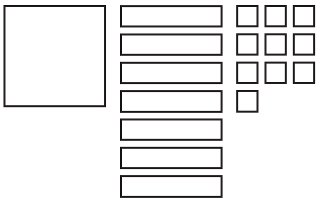
$(3b - 1)(3b + 1)$



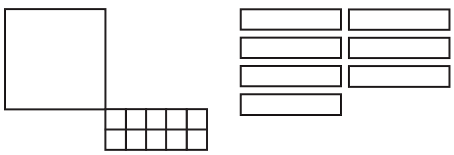
Manipulatives

Algebra tiles can be used to factor trinomials. Use algebra tiles to factor $x^2 + 7x + 10$. Begin by modeling the trinomial.

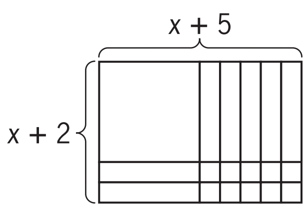
$x^2 \quad + \quad 7x \quad + \quad 10$



Then, arrange the 1's tiles so they form a rectangle. These could be arranged as a 2×5 rectangle or a 1×10 rectangle. Now, arrange the tiles so the lower right corner of the x^2-tile and the upper left corner of the 1's tiles are touching.



Finally, fill in the *x*-rectangles above and to the left of the 1-squares to form a rectangle. All tiles should be used in forming a rectangle. If there are too few *x*-rectangles or if there are *x*-rectangles left over, start over with a different configuration of 1's tiles or try adding zero pairs.



$x^2 + 7x + 10 = (x + 2)(x + 5)$

Use algebra tiles to simplify the following:

1. $x^2 + 2x - 3$

$(x + 3)(x - 1)$

2. $x^2 - 9x + 18$

$(x - 3)(x - 6)$

3. $x^2 - 3x - 28$

$(x - 7)(x + 4)$

4. $x^2 - 10x - 24$

$(x - 12)(x + 2)$

Possible Journal Answers

1. **There are only two pairs of whole number factors for the number 10. The first is one and 10, and the second is two and five. In order for the third term in the trinomial to be negative, one of the factors in the pair has to be negative. The coefficient of the middle term is the sum of the factors. The possible sums are $-2 + 5 = 3$, $-5 + 2 = -3$, $-10 + 1 = -9$, and $-1 + 10 = 9$. Therefore, this trinomial cannot be factored because the coefficient of the second term in the trinomial is not 3, -3, -9, or 9.**
2. **Nicholas used the wrong signs for the factors. Because the middle term of the trinomial is negative and the last term is negative, the larger factor of the last term must be negative. His solution should have been $(x - 6)(x + 3)$. Using the FOIL Method on his solution, the trinomial would need to be $x^2 + 3x - 18$ for Nicholas to be correct.**

3. Because the third term of the trinomial is negative, its factors cannot both be positive or both be negative. One factor must be positive, and one factor must be negative. Therefore, in one binomial factor, the last term must be negative, and in the other binomial factor, the last term must be positive. Because the sum of the factors makes up the coefficient of the middle term of the trinomial and because the middle term is negative, the sum must be negative. The only way to get a negative sum is for the larger addend (in this case, the larger factor) to be negative.
4. Example: $x^2 + 7x + 12$. First, factor the first term of the trinomial, x^2, and place those factors in parentheses: $(x \quad)(x \quad)$. Next, find all the pairs of factors of the last term of the trinomial: $1 \cdot 12$, $2 \cdot 6$, and $3 \cdot 4$. Because the trinomial is of the form $x^2 + bx + c$, where $b > 0$ and $c > 0$, each factor must be positive, so there is no need to list $-1 \cdot -12$, $-2 \cdot -6$, and $-3 \cdot -4$. Now, identify which combination of factors adds up to the coefficient of the middle term of the trinomial. Because the coefficient of the middle term is seven, the factors that work are three and four. The factors can be arranged as $(x \quad 3)(x \quad 4)$ or as $(x \quad 4)(x \quad 3)$. The factored trinomial is $(x + 3)(x + 4)$ or $(x + 4)(x + 3)$.
5. The FOIL Method is used to multiply two binomials. The inverse of multiplication is division. Similarly, factoring a trinomial can be looked at as the inverse of multiplying binomials. Factoring a trinomial involves performing the inverse of each step in the FOIL Method. Instead of multiplying the first terms, F, in each binomial expression to get the first term of the trinomial, the first term of the trinomial must be factored to identify the first terms of each binomial expression. Instead of multiplying the last terms, L, of each binomial, the last term of the trinomial must be factored to identify possible values for the last values in each binomial. Finally, instead of multiplying the outside terms, O, and the inside terms, I, of each binomial expression, the factors of the last term in the trinomial which add up to the value of the coefficient of the middle term must be identified. These factors become the last terms in the binomial expressions.