9.6 teacher notes

$\Delta = .00\pi + \frac{1}{200000}$ $\sqrt{xy}$

Objectives

- Find compositions of two functions.
- Identify functions that are inverses by using composition of functions.

$\Omega \frac{1}{13750}$ $\overline{5\text{-}6} \mid \sqrt{xy}\ \frac{1}{12}\ \Delta$

Vocabulary

Function (Lesson 9-1)
Composition of functions
Domain (Lesson 9-1)
Range (Lesson 9-1)
Inverse functions

Prerequisites

Evaluating functions

Finding the percent of a number

Get Started

- If $f(x) = -x + 2$, find $f(4)$. -2
- If $g(x) = \frac{x}{5}$, find $f(10)$. 2
- If $h(x) = 2x - 1$, find $h(x - 3)$. $2x - 7$

Section 1

Expand Their Horizons

In Section 1, students will evaluate composite functions for particular values. Help students see that they are simply evaluating the functions. It is crucial that students are confident with the material in this section before proceeding to the other sections of this lesson.

Roxy's shoes were originally discounted 60%. This means that she had to pay (100% − 60%) or 40% of the original price. 40% of the original price can be found by multiplying 40% times x or $0.40x$.

The shoes were then discounted an additional 20%. This means that she had to pay (100% − 20%) or 80% of the previous price.

She is paying 80% of 40% of the original cost, \$100. This function can be represented by $f(g(100))$. To evaluate $f(g(100))$, first find $g(100) = 0.4(100) = 40$. Then replace the x in $f(x) = 0.8x$ with 40 to get $f(40) = 0.8(40) = 32$. The original cost of the shoes was \$100, so the final discounted price would be \$32.

Common Error Alert

A common error in working with composition of functions is to evaluate the functions in the wrong order. Instead of $f(g(x))$, the student will find $g(f(x))$. To correct this error, stress that the student should apply the function closest to the variable first.

Find $f(g(0))$. Substitute the 0 into the function closest to it. $g(0) = 4(0) = 0$. $f(0) = \frac{1}{0}$. This is undefined. The denominator of a fraction cannot be equal to zero.

The idea of domain is touched on in the video. Because zero is not in the domain of $f(x)$, $f(g(0))$ is undefined. If the composition $f(g(x))$ were found, it would equal $\frac{1}{4x}$, where $x \neq 0$.

Find $f(g(2))$. Substitute the 2 into the closest function, g. $g(2) = 8$. $f(8) = \frac{1}{8}$.

Find $g(f(2))$. Substitute the 2 into the closest function, f. $f(2) = \frac{1}{2}$. $g\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right) = 2$.

So, $f(g(2)) \neq g(f(2))$.

1 Evaluate $f(g(4))$ and $g(f(4))$ for $f(x) = x + 7$ and $g(x) = 5x$.

$$\begin{aligned} f(g(4)) &= f(5(4)) \\ &= f(20) \\ &= 20 + 7 \\ &= 27 \end{aligned} \qquad \begin{aligned} g(f(4)) &= g(4 + 7) \\ &= g(11) \\ &= 5(11) \\ &= 55 \end{aligned}$$

2 Evaluate $f(g(4))$ and $g(f(4))$ for $f(x) = 3x$ and $g(x) = 3x$.

$$\begin{aligned} f(g(4)) &= f(3(4)) \\ &= f(12) \\ &= 3(12) \\ &= 36 \end{aligned} \qquad \begin{aligned} g(f(4)) &= g(3(4)) \\ &= g(12) \\ &= 3(12) \\ &= 36 \end{aligned}$$

Some students will notice immediately that $f(g(x)) = g(f(x))$ because the two functions are identical.

3 Evaluate $f(g(9))$ and $g(f(9))$ for $f(x) = \sqrt{x}$ and $g(x) = x^2$.

$$\begin{aligned} f(g(9)) &= f(9^2) \\ &= f(81) \\ &= \sqrt{81} \\ &= 9 \end{aligned} \qquad \begin{aligned} g(f(9)) &= g(\sqrt{9}) \\ &= g(3) \\ &= 3^2 \\ &= 9 \end{aligned}$$

There may be some students who will intuitively know the solution to this problem. The square of a square root will give the original value. By the same token, the square root of a square will also give the original value.

Connections

Composition of functions is used quite often in the retail business. In the example in the video, the store offered 60% off the original price and then offered an additional 20% off the 60%-sale price. The store owners assume that the average consumer may think that he or she is getting 80% off the original price and is paying only 20% of the original price. As the lesson showed today, the consumer is actually only getting 68% off the original price and is paying 32% of the original cost.

Additional Examples

1. Evaluate $f(g(3))$ and $g(f(3))$ for $f(x) = x^2$ and $g(x) = x + 1$.

$$f(g(3)) = f(3 + 1) = f(4) = 4^2 = 16$$

$$g(f(3)) = g(3^2) = g(9) = 9 + 1 = 10$$

2. Evaluate $f(g(-3))$ and $g(f(-3))$ for $f(x) = x^2 - 2$ and $g(x) = 3x$.

$$f(g(-3)) = f(3(-3)) = f(-9) = (-9)^2 - 2 = 79$$

$$g(f(-3)) = g((-3)^2 - 2) = g(7) = 3(7) = 21$$

Section 2

Expand Their Horizons

In Section 2, students will find the composition of functions without using specific values. Encourage students to use the same process as in section one. To find $f(g(x))$, substitute $g(x)$ for the variable x in $f(x)$. To find $g(f(x))$, substitute $f(x)$ for the variable x in $g(x)$.

The characters in the video begin by finding $g(☺)$. Although this is not available, it stresses the fact that it does not matter what expression is used. Always replace the original variable with the new expression.

Common Error Alert

Often when students substitute one function for the variable x in the other, they leave the original variable x in place, causing great confusion. Explain to students that $f(x + 1)$ is found the same way as $f(2)$. Always replace the variable with the new expression.

To find $f(g(x))$, replace the x in $f(x) = 0.8x$ with $g(x)$ or $0.4x$. $f(g(x)) = f(0.4x)$. When the x in $0.8x$ is replaced with $0.4x$, the resulting function is $f(0.4x) = 0.8(0.4x) = 0.32x$.

Find $f(g(x))$ if $f(x) = \frac{1}{x}$ and $g(x) = 4x$. $f(4x) = \frac{1}{4x}$. Because the denominator can never equal zero, then $x \neq 0$.

Find $g(f(x))$. $g\left(\frac{1}{x}\right) = 4\left(\frac{1}{x}\right) = \frac{4}{x}$. Since the denominator can never equal zero, then $x \neq 0$.

4 Find $f(g(x))$ and $g(f(x))$ for $f(x) = x + 7$ and $g(x) = 5x$.

$$f(g(x)) = f(5x) = 5x + 7$$

$$g(f(x)) = g(x + 7) = 5(x + 7) = 5x + 35$$

5 This is the same as Guided Notes problem number 3 without a specific value for x. Both $f(g(x))$ and $g(f(x))$ are equal to x.

Additional Examples

1. Find $f(g(x))$ and $g(f(x))$. $f(x) = 2x + 2$ and $g(x) = 4x$.

$$f(g(x)) = f(4x) = 2(4x) + 2 = 8x + 2$$

$$g(f(x)) = g(2x + 2) = 4(2x + 2) = 8x + 8$$

2. Find $f(g(x))$ and $g(f(x))$ for $f(x) = \sqrt{x}$ and $g(x) = x + 3$.

$$f(g(x)) = f(x + 3) = \sqrt{x + 3}$$

$$g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 3$$

Section 3

Expand Their Horizons

In Section 3, students will use the composition of functions to determine if functions are inverse functions.

If the functions are inverses, then both $f(g(x))$ and $g(f(x))$ will equal x. Students may ask, “Are two functions inverses if their compositions are equal but are not equal to x?” The answer is they are not. The compositions of the functions must equal x. Students can think of inverse functions as two functions that undo each other.

In the first example, $f(g(x))$ and $g(f(x))$ both equal x. Therefore, the functions are inverses.

6 Determine if the two functions are inverses of each other. $f(x) = 2x$ and $g(x) = \frac{x}{2}$.

$f(g(x)) = x$. $g(f(x)) = x$. The functions are inverses of each other.

Look Beyond

Students will learn much more about inverse functions as they continue their mathematics' education. One way to find the inverse of a function is to reflect its graph about the line $y = x$. Another way to find the inverse of a function is to switch the x and the y in the equation and solve the new equation for y. Two functions are inverses if for all ordered pairs *(a, b)* in the original function the point *(b, a)* exists in the inverse function.

Additional Examples

1. Determine if the two functions are inverses of each other.
$f(x) = 2x + 3$ $\quad$ $g(x) = \frac{1}{2}x - 3$

$$\begin{aligned} f(g(x)) &= 2(\tfrac{1}{2}x - 3) + 3 \\ &= x - 6 + 3 \\ &= x - 3 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \tfrac{1}{2}(2x + 3) - 3 \\ &= x + \tfrac{3}{2} - 3 \\ &= x - \tfrac{3}{2} \end{aligned}$$

The functions are not inverses because $f(g(x)) \neq g(f(x)) \neq x$.

2. Determine if the two functions are inverses of each other.
$f(x) = 5x$ $\quad$ $g(x) = \frac{1}{5}x$

$$\begin{aligned} f(g(x)) &= 5(\tfrac{1}{5}x) \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \tfrac{1}{5}(5x) \\ &= x \end{aligned}$$

$f(g(x)) = g(f(x)) = x$

The two functions are inverses of each other because $f(g(x)) = g(f(x))$.