9.5 teacher notes

Objectives

- Write and use formulas as functions.
- Given a word problem, write ordered pairs and use them to write functions.
- Use functions to solve problems.

Vocabulary

Function (Lesson 9-1)
Slope-intercept form (Lesson 7-2)
Function notation (Lesson 9-2)
Input (Lesson 9-1)

Prerequisites

Evaluating functions

Graphing linear functions

Solving linear applications

Writing linear equations when given two points

Get Started

- What is a function? A set of ordered pairs no two of which have the same first element.
- Name at least three different types of functions. linear function, constant function, absolute value function, (other answers may apply)
- What are at least two ways functions can be used in the real world? Answers will vary, but may include carpentry, business, travel schedules, pay schedules, etc.

Section 1

Expand Their Horizons

In Section 1, students will use functions that involve formulas and linear relationships to solve real-world situations. It is very important that the students see the connection between algebra and the real-world.

In the opening dialogue, the characters are discussing functions and how they can be used to model situations where one quantity depends on the value of one or more other quantities. Ferd relates this to a situation in which his illness depended on how many bags of chips he ate. Ask students if they can name other situations that can be described using functions.

Many students have had experience painting in one form or another. Ask for input allowing students to describe how they decided how much paint was necessary to complete the job. How many students have ever started a job and then run out of paint before the job was completed?

This shed is eight feet long by eight feet wide by eight feet tall. By figuring the area to be painted without windows and doors, the problem is simplified considerably. The area of one wall can be found by $A = s^2$. Because the area is dependent upon the length of a side, the area is a function of the length of the side. This formula can then be written as $A(s) = s^2$.

It may be helpful at this time to make a sketch of a shed or to use a model of a cube to show students that a shed with a flat roof will have five congruent sides.

Common Error Alert

Rounding errors often occur in application problems. Students must pay particular attention to the question being asked to determine which direction to round.

The area of the shed to be painted is $A(s) = 5s^2$. Because one can of paint will cover approximately 350 ft^2, the amount of paint needed can be found by $c(s) = \frac{5s^2}{350}$. This is approximately 0.914 cans of paint. At this time it is appropriate to discuss rounding methods with the students. What if the answer had been 0.213 cans of paint? Would the answer still have rounded to one? Yes. The painter must have enough paint to finish the job. All answers will round to the next larger whole number.

Common Error Alert

When students are listing ordered pairs they will often leave out either the $(x, 0)$ or the $(0, y)$ ordered pair. Using a table to list all possibilities may help eliminate this error.

Explain to students that function applications can be solved in many different ways. They can be solved using formulas, tables, charts, graphs, and equations.

Ferd bought a mower for $225. He charges an average of $20 per lawn. He will not show any profit until the total amount owed on the mower is paid. Therefore, the function will be $p(x) = 20x - 225$.

He will break even when his revenue is zero.

1 h = the number of hours Roxy works; $E(h)$ = the earnings for a given number of hours. If she makes nine dollars an hour, then $E(h) = 9h$. Substituting eight for h, gives $E(8) = 9(8) = \$72$. Students should write a sentence describing their solution, such as, "Roxy makes $72 if she works 8 hours."

Common Error Alert

To find the break-even point, students will often substitute zero for x instead of $p(x)$. Writing what each variable represents may eliminate this error. In the previous example, they would write "x = number of yards mowed, $p(x)$ = profit from a given number of yards."

2 Students may see these problems as being too simple and may resist writing functions because they can figure the answer mentally. Explain that they must learn to write the simple functions so that they will have the skills necessary to solve problems that they cannot work mentally. This problem could have also been solved using a table.

x	$s(x)$
0	6
1	5
2	4
3	3
4	2
5	1
6	0

Additional Examples

1. Write an equation that could be used to find the area of the walls of a square room. Then, find the area of the walls if one side is 12 feet.

The area of one wall is $A(s) = s^2$. There are four walls in the room, so the area of the walls of the room is $A(s) = 4s^2$.
$A(12) = 4(12^2) = 576 \text{ ft}^2$

2. Sam is working as a waiter. He makes $100 a week in salary. He also averages $20 a night in tips. Write an equation for the function that describes how much money Sam will make in one week after working *x* nights.

$f(x) = 100 + 20x$

Section 2

Expand Their Horizons

In Section 2, students will write linear functions from two points on a line. They may use either the slope-intercept form or the point-slope form to find the linear function.

Because the target blood pressure is dependent on the person's age, the target blood pressure is a function of age. The independent variable is age. The dependent variable is blood pressure. Find the slope using the slope formula. In this case the slope is $\frac{1}{2}$.

Next, find the slope-intercept form of the equation. If preferred, the point-slope form may be used.

$$y - y_1 = m(x - x_1)$$
$$y - 115 = \tfrac{1}{2}(x - 10)$$
$$y - 115 = \tfrac{1}{2}x - 5$$
$$y = \tfrac{1}{2}x + 110$$

Written in function notation, this equation becomes $f(x) = \frac{1}{2}x + 110$.

3 The pattern is linear. After finding the slope, use either the point-slope form or the slope-intercept form to find the function. $m = 210$. $y - y_1 = m(x - x_1)$

$$y - 0 = 210(x - 0)$$
$$y = 210x$$

Because the function is dependent upon time, $x = t$ and $y = f(t)$.

$$f(t) = 210t$$
$$f(30) = 210(30)$$
$$f(30) = 6{,}300$$

Students may be hesitant to use the point (0, 0) for x_1 and y_1. Because this point is included in the line, it is acceptable to use the point.

Additional Examples

1. **The cost of concert tickets for a particular concert can be modeled by a linear function. The charge for one concert ticket is \$45. The charge for two tickets is \$75. Write the equation for the function and use the function to find the cost of three tickets.**

$$m = \frac{75 - 45}{2 - 1} = 30$$
$$f(x) = mx + b$$
$$45 = 30(1) + b$$
$$15 = b$$
$$f(x) = 30x + 15$$
$$f(3) = 30(3) + 15$$
$$f(3) = 105$$

It will cost \$105 for three tickets.

2. **Tracy travels 105 miles in 2.5 hours. At the same rate, she travels 298.5 miles in 7 hours. Write the equation for the function and use the function to find the distance Tracy will travel in 10 hours.**

$$m = \frac{298.5 - 105}{7 - 2.5} = 43$$
$$f(x) = mx + b$$
$$105 = 43(2.5) + b$$
$$105 = 107.5 + b$$
$$-2.5 = b$$
$$f(x) = 43x - 2.5$$
$$f(10) = 43(10) - 2.5$$
$$f(10) = 427.5$$

Tracy will travel 427.5 miles in 10 hours.