9.4 teacher notes

Objectives

- Graph linear functions from slope-intercept form.
- Graph constant functions.
- Graph absolute value functions.
- Graph piecewise functions.

Vocabulary

Relation (Lesson 9-1)
Domain (Lesson 9-1)
Range (Lesson 9-1)
Identity function
Constant function (Lesson 9-1)
Absolute value function
Piecewise function

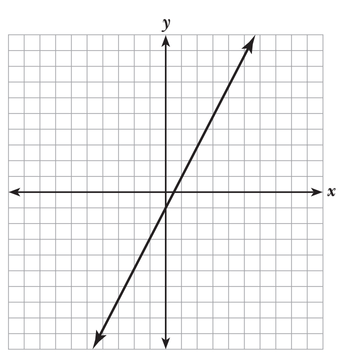
Prerequisites

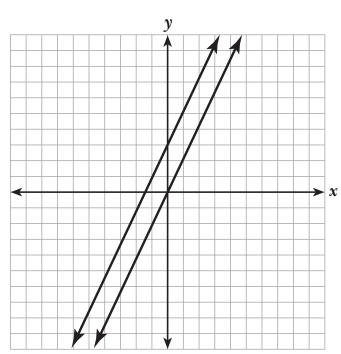
Graphing linear equations

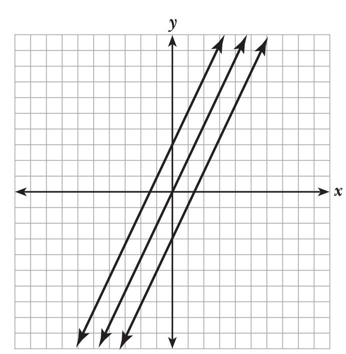
Graphing equations of two variables by plotting points

Get Started

- Graph $y = 2x$. see graph below
- Graph $y = 2x + 3$ on the same coordinate plane. see graph below
- Graph $y = 2x - 3$ on the same coordinate plane. see graph below
- How are these graphs related? The graphs are parallel lines.
  The graph of $y = 2x + 3$ is 3 units above the graph of $y = 2x$.
  The graph of $y = 2x - 3$ is 3 units below the graph of $y = 2x$.







Section 1

Expand Their Horizons

In Section 1, students will be graphing linear functions that are written in function notation. They will learn special names for some of these functions. The function $f(x) = x$ is the identity function. A horizontal line is a constant function.

The first function graphed is the identity function. Help students remember this name by showing them that each output value is identical to its corresponding input value.

The next example is first written as $y = 2x + 3$ and then written as $f(x) = 2x + 3$. This will reinforce the students' familiarity with function notation. Encourage students to use function notation in their work.

It may be necessary to pause the video for the next example, $f(x) = -x$. Let students find the slope, –1 and the y-intercept, 0.

Have students compare the graphs of $f(x) = x$ and $f(x) = -x$ as they compare the equations. Point out that there are lines with opposite slopes and the same y-intercept; yet, they are perpendicular.

If $f(x) = -x$, find $f(-4)$ by substituting –4 for x. $f(-4) = -(-4) = 4$.

1 Graph the linear function $f(x) = \frac{x}{3}$. Students may want to rewrite this equation in slope-intercept form as $f(x) = \frac{1}{3}x + 0$. The line has a slope of $\frac{1}{3}$ and a y-intercept of 0. $f(-6) = \frac{-6}{3} = -2$.

The next type of function is the constant function. Students may not recognize the constant function written using function notation. If not, rewrite the equation for them as $y = 3$. Why is this function called the constant function? No matter what value is given for x, the value for y is always constant.

Point out that the graph of the constant function $f(x) = 0$ is the x-axis.

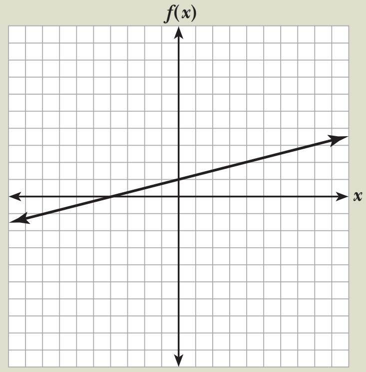
2 Graph the constant function $f(x) = -5$. This is the same as $y = -5$. It is a horizontal line.

Common Error Alert

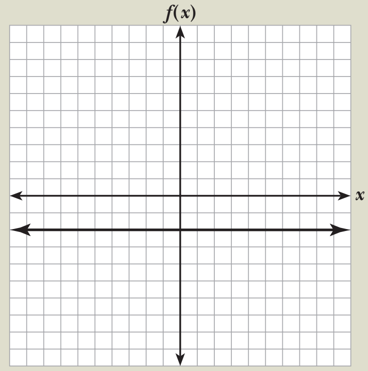
Some students may try to represent vertical lines using function notation. Vertical lines may not be expressed using function notation because vertical lines are not functions. To further illustrate this point, draw a vertical line and list some points on the line. Students will see that the set of points do not satisfy the definition of a function.

Additional Examples

1. Graph the linear function $f(x) = \frac{x}{4} + 1$.
Rewrite $\frac{x}{4}$ as $\frac{1}{4}x$ so that the equation is $f(x) = \frac{1}{4}x + 1$. The slope is $\frac{1}{4}$. The y-intercept is 1.



2. Graph the constant function $f(x) = -2$.



Section 2

Expand Their Horizons

In Section 2, students will be graphing absolute value functions. Because the absolute value of a function is always positive or zero, the graph of $f(x) = |x|$ has no negative $f(x)$ or y values. Therefore, the graph does not exist in the third and fourth quadrants.

The graph of the absolute value function looks like a V.

Students who learn kinesthetically may benefit from an exercise showing the changes in the absolute value function when values are added and subtracted. Have all students stand and represent the graph of $y = |x|$ by holding their arms above their heads to form a V. Write $f(x) = |x - 3|$ on the board. Students should take three small steps to the right. Write $f(x) = |x + 3|$ on the board. Students should take three small steps to the left. Write $f(x) = |x| + 3$ on the board. Students can take three small steps forward to represent moving up three units. Write $f(x) = |x| - 3$ on the board. Students can either squat down or take three small steps back to represent moving down three units.

There are many different parent graphs. If students learn the parent graph for the absolute value function, then they can use the translation rules to graph many different absolute value functions. It is much simpler to learn the parent graph than to plot points each time.

3 Graph the function: $f(x) = |x + 3| - 6$. This is the graph of an absolute value function. The parent function is translated three units to the right and six units down. The new vertex is (3, –6).

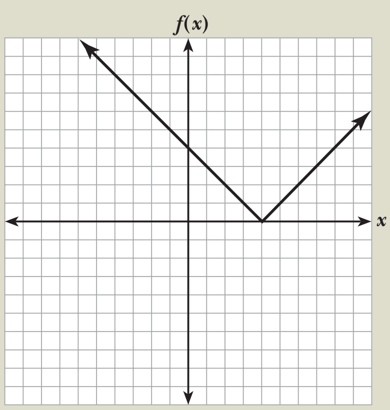
Common Error Alert

Many students will move the wrong direction when graphing a function such as $f(x) = |x - 3|$ or $f(x) = |x + 3|$. If necessary, plot several points to show students that these graphs actually move the opposite direction of the sign shown.

Additional Examples

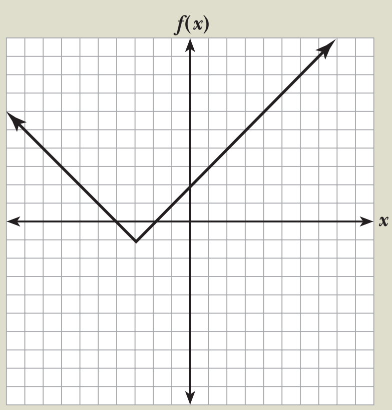
1. Graph the function: $f(x) = |x - 4|$.

This equation will move the parent graph four units to the right. The new vertex is (4, 0).



2. Graph the function: $f(x) = |x + 3| - 1$.

This equation will move the parent graph three units to the left and one unit down. The new vertex is (–3, –1).



Section 3

Expand Their Horizons

In Section 3, students will graph piecewise functions. These are functions that are defined differently for different sections of the graph. These functions never overlap. This is shown by the proper use of open and closed circles on the graph.

Common Error Alert

Students may use either closed circles when an open circle is appropriate or an open circle when a closed circle is appropriate. Remind students that a closed circle indicates that the point is considered part of the solution. An open circle indicates that the point is not considered part of the solution.

Piecewise functions should be graphed one section at a time. For the first example, graph $f(x) = -x - 2$. This line has a slope of negative one and a y-intercept of negative two. Graph this line, but stop at the y-axis where x is equal to zero. Put an open circle on the y-axis because of the less than symbol. Be sure students are paying attention to the direction of the inequality symbol. Next, graph the line $f(x) = \frac{x}{3} - 2$. This line has a slope of $\frac{1}{3}$ and a y-intercept of negative two. Graph this line on the same coordinate plane, beginning at the y-axis and continuing to the right. The two lines will intersect at the point (0, –2). This line will have a closed circle at that point. Although this function is continuous, its slope changes at the y-axis.

Look Beyond

As students continue their mathematics' education, they will be graphing many different types of functions. For example, they will graph quadratic functions, trigonometric functions, and step functions. Each of these different types of functions can be used to model real-world situations.

Graph $f(x) = -3$ for $x < -1$. This graph will begin with an open circle at the point (–1, –3) and form a horizontal line going infinitely to the left. The next part of this function, $f(x) = x + 2$ for $-1 \leq x \leq 3$, will be a line with a slope of 1 and a y-intercept of 2 beginning at the point (–1, 1) and ending at the point (3, 5). Both of these points will have closed circles. The third part of this function, $f(x) = 5$ for $x > 3$, will be a horizontal line beginning at the point (3, 5) and going infinitely to the right. The point (3, 5) will be an open circle. Because this function seems to jump from section to section, it is said to have jump discontinuity.

Connections

Graphs are used to show trends and are often used in modeling business activity. Businesspeople make and interpret graphs to understand market trends and use graphs to make decisions with respect to marketing trends, hiring employees, and production schedules.

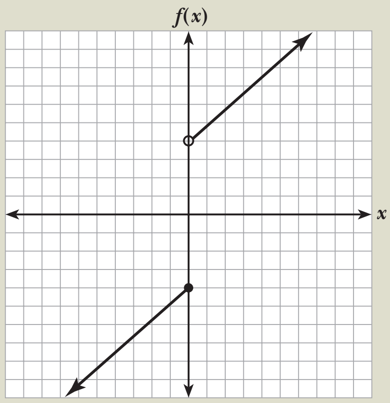
4 This graph consists of parts of three different lines. Graph the first part of the function, $f(x) = x + 1$ for $x \leq -3$. This is part of the line with a slope of 1 and a y-intercept of 1. It has a closed circle at the point (–3, –2) and extends infinitely down to the left. The next part of the function, $f(x) = 2x$ for -3 , $x \leq 1$, is part of the line with a slope of 2 and a y-intercept of zero. It connects points (–3, –6) and (1, 2), with an open circle at (–3, –6) and a closed circle at (1, 2). The final section of this graph, $f(x) = 4$ for $x > 1$, is part of a horizontal line. It begins with an open circle at the point (1, 4) and extends infinitely to the right.

Additional Examples

1. Graph the following piecewise function:

$$f(x) = \begin{cases} x - 4, & x \le 0 \\ x + 4, & x > 0 \end{cases}$$

Graph the first section, $f(x) = x - 4$, beginning with a closed circle at the point (0, –4). Graph the second section with an open circle beginning at the point (0, 4).



2. Graph the following piecewise function:

$$f(x) = \begin{cases} -x, & x \le -2 \\ \frac{x}{2} + 3, & -2 < x \le 4 \\ 7, & x > 4 \end{cases}$$

Graph the first section, $f(x) = -x$ for $x \le -2$, beginning at the point (–2, 2) with a closed circle. Graph the next section, $f(x) = \frac{x}{2} + 3$ for $-2 < x \le 4$, beginning at the point (–2, 2) with an open circle and ending at the point (4, 5) with a closed circle. The last section is a horizontal line beginning at the point (4, 7) with an open circle.