9.3 teacher notes

Objectives

- Write a function rule for a linear pattern.
- Write a function rule for a nonlinear pattern.

Vocabulary

Function (Lesson 9-1)
Slope (Lesson 7-2)
Slope-intercept form (Lesson 7-2)

Prerequisites

Evaluating functions

Finding the equation of a line when given a point and the slope

Get Started

- Find the next three numbers in the following sequence:
  2, 5, 8, 11, ___, ___, ___. 14, 17, 20
- How were these numbers found? Add 3 to the previous number
- What is an expression for the next term after the n^{th} term in the sequence? $n + 3$

Section 1

Expand Their Horizons

In Section 1, the students will find a pattern in a set of numbers and write a rule that tells what is being done to the input number to get the output number. Then they will use this rule to evaluate the function for various input values.

The comet can be seen every seven years. Students may need to see that the first time Lizzie will be able to see the comet will be in $7(1) = 7$ years, the second in $7(2) = 14$ years, the third in $7(3) = 21$ years, etc. From these examples, they can determine that the rule is $f(x) = 7x$.

The candle's original height is 25 cm. Each hour the candle becomes one cm shorter. Ask students, "What operation can best be used to describe shrinking by one cm?" Help them understand that the answer to this question is subtraction. The height decreases one cm every hour. Therefore, the rule is $f(x) = 25 - x$. If students are still having difficulty with this equation, show them the height of the candle at the end of each hour. After one hour the candle is 24 cm. After two hours the candle is 23 cm. After three hours the candle is 22 cm. These results can be found by $25 - 1$, $25 - 2$, and $25 - 3$, respectively.

Common Error Alert

Students sometimes will look at the total number of cm that the candle has shrunk after each hour and determine an incorrect rule such as $f(x) = x$ or $f(x) = 25 + x$. Steer them away from this erroneous thinking by helping them concentrate on the height of the candle instead of the number of cm lost.

Remind students once again that $f(x)$ is taking the place of y and represents the height of the candle. If the height is zero, then $f(x)$ is zero. Discuss with students what it means if the height of the candle is zero. They should realize that when the height of the candle is zero, the candle has burned completely. To determine when the height of the candle will be zero, solve the equation $0 = 25 - x$. The candle will be gone after 25 hours.

1 Each input must be divided by 3 to find the corresponding output. Remind students that a fraction is one way of representing division. $\frac{5}{3}$ means the same as 5 divided by 3.

2 Lead the students to the conclusion that this table represents a constant function. No matter what value is substituted for x, the output is zero. Therefore, the rule is $f(x) = 0$.

Although the output values in the next example increase by an increment of 9, the output values are not multiples of nine. Therefore, the rule is not $f(x) = 9x$. Upon closer inspection, students should see that each value is exactly 100 more than the previous rule would allow. The rule for this function is $f(x) = 9x + 100$.

The pattern for the next set of ordered pairs is not obvious at first. The characters in the video use a scatterplot to determine the rule. Be sure students understand that a scatterplot is made by graphing the ordered pairs that make up the set. These ordered pairs appear to lie in a straight line. Students should check at least two pairs of points to determine that the slope between any two ordered pairs is the same slope and, therefore, the points do lie in a line. At this time students should use their knowledge of writing linear equations to find the rule for this set of points.

Students often see the different concepts of algebra as being unconnected bits of information that must be used only for a particular chapter. Help students see how the knowledge that they gained previously in learning to write linear equations when given a point and a slope is now being used to write rules for functions.

3 First, have students plot the ordered pairs represented in the table. Although the points appear to be linear, have students check at least two pairs of the points to see that they get the same slope. Although the video uses the slope-intercept form of the equation to find the equation, many students may want to use the point-slope form:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= -1(x - 2) \\ y - 1 &= -x + 2 \\ y &= -x + 3 \end{aligned}$$

4 Some students may be able to intuitively determine that the graph for this set of ordered pairs is linear by studying the pattern of the coordinates. As the *x*-coordinates increase by 3, the *y*-coordinates increase by 2. Because this pattern is consistent, the function is linear with a slope of $\frac{2}{3}$.

Additional Examples

1. Write a function for the ordered pairs shown in the table.

x	*y*
–1	3
2	–6
4	–12
5	–15

Each input value is multiplied by –3 to obtain an output. The rule is $f(x) = -3x$.

2. Find a function containing the following ordered pairs: (1, 4), (3, 2), (5, 0), (7, –2).

These points appear to be linear. The slope between the first two points is –1. The last two points also have a slope of –1. Use the point-slope form of an equation and the first point to find the equation of the line.

$$\begin{aligned} y - 4 &= -1(x - 1) \\ y - 4 &= -x + 1 \\ y &= -x + 5 \end{aligned}$$

Now check to see that each ordered pair makes the equation true. Finally, write the function $f(x) = -x + 5$.

Section 2

Expand Their Horizons

In Section 2, students will be finding rules for nonlinear functions. It may be necessary for students to try several ordered pairs before determining a rule for the set of ordered pairs.

5 Each value of the output is the square root of the corresponding value of the input. Students may see this as "the output, squared, is equal to the input." Lead them to the conclusion that this is the same as

Common Error Alert

Students may have difficulty recognizing patterns involving powers. They may use addition and multiplication to find a rule that will fit some of the numbers and ignore the numbers that do not work with the rule that they have chosen. Caution students to be sure to include all of the ordered pairs when determining the rule.

"the square root of the input is equal to the output." Also remind students that when the square root symbol is used, it always stands for the nonnegative square root of the number.

6 This example may be a little more difficult for students to see. If students are having a great deal of difficulty, help them follow Newt's reasoning by noticing that each value in the output is a perfect square. Then, help them to notice that $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, and to use this knowledge to see that the rule is $f(x) = (x + 1)^2$.

Look Beyond

In calculus, students will study a special type of function called a sequence. They will be asked to discover patterns in sequences and to describe rules for generating those sequences.

Connections

Astronomers look for patterns when they study the movement of different celestial bodies. After careful observation they will formulate rules to describe those movements so that they can predict the location of the celestial bodies at any given time.

Common Error Alert

Students may be inclined to square each term of the expression instead of squaring the entire expression. Have students substitute values for x to see that they must square the entire expression.

It may be necessary to review slope and slope-intercept form with the students. To do this, refer to Module 7, Lesson 2: *Graphing Linear Equations of Two Variables.*

Additional Examples

1. Write a function for the input/output table.

Input	Output
–2	7
1	4
2	7
3	12

Since the same output is paired with both 2 and –2, the output is probably found by using x^2. $7 = (-2)^2 + 3$. $4 = (1)^2 + 3$. $7 = (2)^2 + 3$. $12 = 3^2 + 3$. The function is $f(x) = x^2 + 3$.

2. Write a function for the set of ordered pairs {(0, 0), (1, –1), (2, –8), (3, –27)}.

Each output is the opposite of the cube of the input. The function is $f(x) = -x^3$.