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9.1 teacher notes

Objectives

- Represent a relation in set notation, a table, a mapping diagram, a coordinate graph, an equation, and a relation rule.
- Determine domain and range of a relation.
- Determine whether a relation is a function.

Vocabulary

Relation
Domain
Range
Real number (Lesson 1-1)
Function
Constant function
Vertical line test

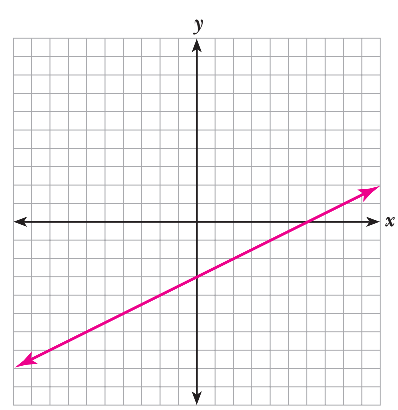
Prerequisites

Graphing ordered pairs

Graphing linear equations

Get Started

- Graph the line $y = \frac{1}{2}x - 3$.
- Find five points on the line. Answers may vary. (–2, –4), (0, –3), (2, –2), (4, –1), (6, 0)
- List the set of first elements or x's of the set of points that are found. {–2, 0, 2, 4, 6}
- List the set of second elements or y's of the set of points that are found. {–4, –3, –2, –1, 0}



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Section 1

Expand Their Horizons

In Section 1, the students find the domain and the range of various relations that are represented by sets of ordered pairs, mapping diagrams, tables, equations, and graphs. The sets will be both finite and infinite.

In the first example, New York is listed twice in the set of ordered pairs. Point out to students that New York is only listed once in the domain. Stress the concept that a set can be any group of objects. The elements of a set do not have to be numbers.

In the second example, although the set {1, 9, 8} is equivalent to the set {1, 8, 9}, have students get in the practice of listing the domain and range in order from least to greatest as it is done in the video.

1 Find the domain and range of the relation shown. Have students make a list of the ordered pairs represented by the points on the graph: (0, 3), (–5, 3), (2, 3), and (4, 3). Then, have them select the first elements of the ordered pairs as the domain and the second elements of the ordered pairs as the range. See that students list the elements in order from least to greatest. Also, remind students, that in the range, 3 should only be listed once.

The next example is the line whose equation is $y = x + 2$. The line consists of an infinite set of points. The domain {x: x is a real number} can be read, "the set of all x such that x is a real number." This same set can be written as {x: $x \in \Re$}. The symbol $\in$ is the Greek letter epsilon and in mathematics means, "is an element of." When this notation is used, the set is read, "the set of all x such that x is an element of the set of real numbers."

The next example uses the graph $y = |x|$. Help students use the graph to see that no matter what the x-coordinate is, the corresponding y-coordinate is always a nonnegative value.

2 In the mapping diagram, both 1 and 4 are mapped to 4. Students should write both ordered pairs but should include the 4 only once in the range.

Additional Examples

1. Find the domain and range of the relation S = {(–3, 2), (2, 0), (–3, 5), (0, 1)}.

Domain = {–3, 0, 2}
Range = {0, 1, 2, 5}

2. Let the domain of the relation given by the equation $y = 2x + 4$ be {–2, 0, 2}. Find its range.

$y = 2(-2) + 4$	$y = 2(0) + 4$	$y = 2(2) + 4$
$y = -4 + 4$	$y = 0 + 4$	$y = 4 + 4$
$y = 0$	$y = 4$	$y = 8$

Range = {0, 4, 8}

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Section 2

Expand Their Horizons

In Section 2, students will use what they have learned about finding the domain and the range to determine if a relation is a function and if a function is a one-to-one mapping. They will then apply this knowledge and will learn to use the vertical line test.

Because New York is mapped to both Yankees and Mets, the first example is not a function.

In the same manner, because zero is mapped to both one and eight, the second example is not a function.

Connections

Functions are important in many businesses, especially when finding costs or planning prices. The U.S. Postal Service uses a function to determine the amount of postage for sending letters and packages. Sometimes the postage depends on the weight of the letter or package. If the formula used were not a function, then two first-class letters weighing the same amount could have two different amounts of postage.

For the same reasons, airfare, car rental rates, long distance charges, and recycling payments are computed using functions.

One way of describing a one-to-one mapping that deals with ordered pairs is to say that each x is paired with exactly one y, and each y is paired with exactly one x.

The next example dealing with relation g could be confusing for the students. Remind students that in a function the first elements, or x's, cannot be the same, but the second elements, or y's, can be. Relation g is a function. It is not, however, a one-to-one mapping.

A horizontal line represents a constant function. No matter what the value of x is, the value of y is always constant.

Any line that is not a vertical line represents a function. A vertical line has all its first elements the same. It does not represent a function.

Common Error Alert

Some students will check every x-value to see if there is a unique corresponding y-value, instead of checking every y-value to see if there is a unique corresponding x-value. Use the vertical line test to show that for a function, each y-coordinate has a unique x-coordinate.

The graph of the function $s(x) = x^2$ is a parabola with its vertex at the origin. Students can graph these points and use the vertical line test to determine that this relation is a function.

Many students will find that a vertical line test is the easiest way for them to determine if a graph represents a function. As long as there is no vertical line that intersects the graph in more than one point, the graph represents a function.

3 Have students take the time to actually draw some vertical lines through the graphs on their Guided Notes sheet. Help them to see that any vertical line drawn through graphs *a* and *c* would intersect these graphs in only one point.

Look Beyond

As a student continues his or her mathematics education, he or she will be required to write equations in function notation. An equation that is now written as $y = 2x + 1$ will sometimes be written as $f(x) = 2x + 1$. $f(x)$ is read "f of x." It stands for the value of the function with respect to the variable x. To evaluate the function $f(x) = 2x + 1$ for $x = 3$, substitute 3 for x. Write $f(3) = 2(3) + 1 = 7$.

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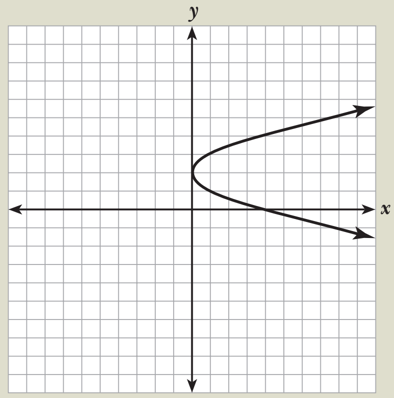
Additional Examples

1. Use the vertical line test to determine if the given graph represents a function. Justify this answer.

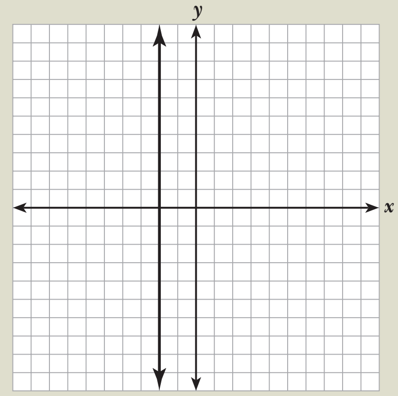


The relation is not a function. A vertical line can intersect the graph in two points. Therefore, one *x*-coordinate is mapped to more than one *y*-coordinate.

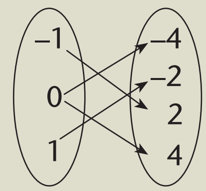
2. Which of the following relations are functions? Write *Yes* if it is a function or *No* if it is not a function. Then give a reason for this choice.

a) $N = \{(0, 1), (-3, 4), (-2, 4), (5, 1)\}$

b)



c)



a) Yes, all first elements of the set are unique.
b) No, the graph is a vertical line. All first elements are the same.
c) No, the 0 is mapped to both 4 and –4.

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