8.4 teacher notes

Objectives

- Transform the equation into slope-intercept form when it is given in standard form.
- Graph an equation given in standard form.
- Identify the effects of parameter changes on the appearance of graphs.

Prerequisites

Writing equations of lines given the slope and the *y*-intercept

Writing equations of lines given a point and the slope or two points

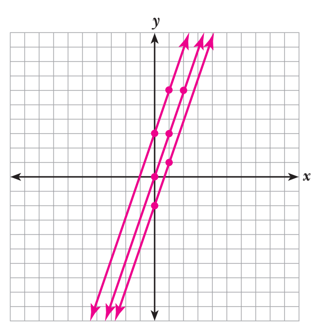
Finding slope

Vocabulary

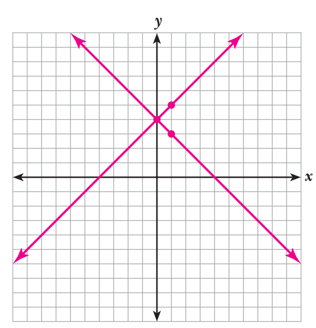
Parameter
Parallel lines (Lesson 8-1)
Perpendicular lines (Lesson 8-1)
Standard form of a linear equation
Slope (Lesson 7-2)
y-intercept (Lesson 7-2)
Negative reciprocals (Lesson 8-1)
Slope-intercept form (Lesson 7-2)
Point-slope form (Lesson 8-3)

Get Started

- Graph $y = 3x$, $y = 3x + 3$ and $y = 3x - 2$ on the same coordinate plane.
- How are these graphs related? They are parallel.



- Graph $y = x + 4$ and $y = -x + 4$ on the same coordinate plane.
- How are these graphs related? They intersect at their *y*-intercepts and are perpendicular.



Section 1

Expand Their Horizons

The parameters of an equation are the quantities that help determine its appearance. The parameters of a linear equation in slope-intercept form are the slope and the *y*-intercept. By manipulating the parameters of linear equations and noting the changes that take place in the graphs, students may improve their understanding of linear equations. They will see for themselves that lines with the same slope are parallel. Lines with different slopes but the same *y*-intercept will all intersect at the same point, the point containing the *y*-intercept.

As the lines appear on the graph, have students work along with the characters in the video. By now students should be able to find the equations of the lines. If they are still struggling in this area, pause the video, help them find the slopes and *y*-intercepts of the lines, and use these values to write the equations.

In the first example, the parameter m of the lines is the same. Therefore, the lines are parallel. The parameter b, however, is different for each line. Therefore, the lines intersect the *y*-axis at different points.

Connections

Businessmen must study the parameters of their businesses carefully. Changing one or more parameters of a business could cause the business to become much more financially successful, or it could cause its downfall. Some of the parameters of a business are its number of employees, its location, the quantity of goods produced, the cost of raw materials, and the price of the final product. Are there other parameters?

Common Error Alert

Students may confuse slope and *y*-intercept. Review the slope-intercept form, stressing that m is slope and b is *y*-intercept.

In the next example, the lines have the same *y*-intercept but different slopes. Use this set of lines to examine how slope affects the appearance of a line. To have a negative slope, one has a slope of zero, and one has a positive slope. Help students notice that the lines with negative slopes rise to the left, the line with a slope of zero is horizontal, and the line with positive slope rises to the right. Also, have students compare the lines with slopes of –2 and $\frac{-2}{3}$. These lines both rise to the left, but the line with a slope of –2 is steeper than the line with a slope of $\frac{-2}{3}$. The greater the absolute value of the slope, the steeper the line.

The next pair of lines has slopes that are negative reciprocals. These lines are perpendicular. The fact that these lines have the same *y*-intercept is not relevant to the fact that the lines are perpendicular. To reinforce this awareness, have students graph a pair of lines whose slopes are negative reciprocals, and whose *y*-intercepts are different. An example would be the graphs of the equations $y = 2x - 2$ and $y = -\frac{1}{2}x + 4$. These lines are perpendicular and have different *y*-intercepts. For problems 1-3, provide students with grid paper so they can graph the equations and compare.

1 Given $y = -\frac{1}{4}x - 2$. Determine the resulting equation when the *y*-intercept is increased by 6. Compare the graphs. Adding 6 to the *y*-intercept gives the equation $y = -\frac{1}{4}x + 4$. This equation has the same slope as the original equation but is six units higher.

2 Given $y = -\frac{1}{4}x - 2$. Determine the resulting equation when the slope is multiplied by –16. Compare the graphs. Multiplying the slope by –16 gives $y = 4x - 2$. This equation has the same *y*-intercept as the original equation, but its graph rises to the right and is much steeper.

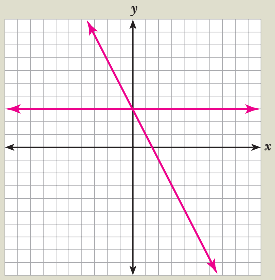
3 Given $y = 3x - 4$. Divide the slope by –6. The resulting line will rise to the left and be less steep.

Additional Examples

1. Given $y = -2x + 3$. Determine the resulting equation when the slope is increased by 2. Compare the graphs.

$$y = (-2 + 2)x + 3$$
$$y = 0x + 3$$

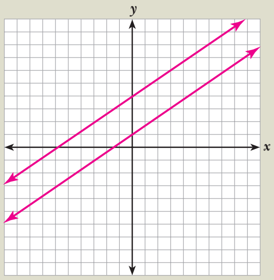
The lines have the same *y*-intercept. The resulting line is horizontal.



2. Given $y = \frac{2}{3}x + 1$. Determine the resulting equation when the *y*-intercept is increased by 3.

$$y = \tfrac{2}{3}x + 1 + 3$$
$$y = \tfrac{2}{3}x + 4$$

The lines are parallel. The resulting line intersects the *y*-axis 3 units higher than the original line.



Section 2

Expand Their Horizons

In Section 2, students are introduced to the Standard Form of Linear Equations. In this course, standard form is written as $Ax + By = C$. In some texts, standard form is written as $Ax + By + C = 0$. Either form is acceptable.

Standard form is often used for a linear equation. However, slope-intercept form is usually better for graphing purposes. To convert an equation in standard form to slope-intercept form, solve the equation for *y*.

$$Ax + By = C$$
$$By = C - Ax$$
$$y = \frac{(C - Ax)}{B}$$
$$y = \frac{C}{B} - \frac{Ax}{B}$$
$$y = \frac{-A}{B}x + \frac{C}{B}$$

Slope can be found directly from the standard form $m = \frac{-A}{B}$. The *y*-intercept can be found directly from the standard form $b = \frac{C}{B}$. There is no need to memorize these facts, because students should be able to transform an equation from standard form to slope-intercept form.

In the first example, the equation $3x + 6y = 12$ is given. To transform it into slope-intercept form, solve it for *y*. Help the students through this process. The result is $y = -\frac{1}{2}x + 2$.

Common Error Alert

Students may make sign errors when they divide both sides of an equation by a negative number. Emphasize the importance of checking their work for this mistake.

In the next example, $3x + 6y = 12$ is given again with the directions to determine the resulting equation if the y-intercept is decreased by 2. But, as Lizzie points out, the equation $3x + 6y = 12$ in slope-intercept form is $y = -\frac{1}{2}x + 2$. The y-intercept is 2. So, the new y-intercept is 0, and the resulting equation is $y = -\frac{1}{2}x$.

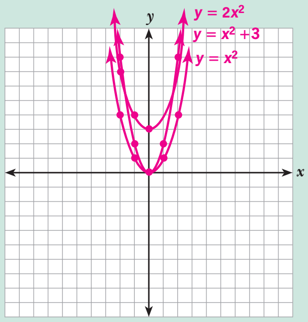
In the next example the same equation is given a third time, and the class is asked to find a line with the same y-intercept and twice the slope of the given line. Write the equation in slope-intercept form, $y = -\frac{1}{2}x + 2$. The slope is $-\frac{1}{2}$. When the slope is doubled, the equation of the new line is $y = -x + 2$. This line is steeper than the original line.

4 The equation is given in standard form, and students are asked to change both parameters. Change the equation to slope-intercept form $y = 3x - 6$. Multiply the slope by one-third and add four to the y-intercept: $y = \left(\frac{1}{3}\right)(3)x - 6 + 4$, or $y = x - 2$. Students can see from the graphs that this line is less steep than the given line and is translated four units up.

Look Beyond

Students will learn about families of graphs in future studies. The graphs in a family of graphs have a common characteristic that differentiates that family from other families. For example, the graphs of $y = x^2$, $y = x^2 + 3$, and $y = 2x^2$ are all in the same family because they are parabolas.

The graph of $y = x^2$ is called the parent graph of this family. The graph of $y = x^2 + 3$ is the parent graph translated 3 units up, and the graph of $y = 2x^2$ is the parent graph made "narrower."

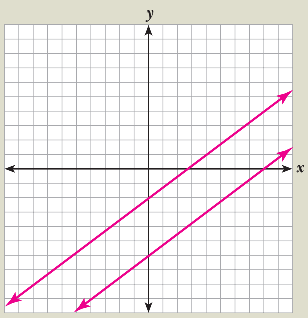


Additional Examples

1. Find an equation of the line with the same slope as the line $3x - 4y = 8$ but with the y-intercept decreased by four.

$$3x - 4y = 8$$
$$-4y = 8 - 3x$$
$$y = \frac{8 - 3x}{-4}$$
$$y = -2 + \frac{3}{4}x$$
$$y = \frac{3}{4}x - 2$$
$$y = \frac{3}{4}x - 2 - 4$$
$$y = \frac{3}{4}x - 6$$

The lines have the same slope. The resulting line is translated down four units.



2. Find the slope and y-intercept of $x - 2y = 4$. Find an equation of the line whose slope is $\frac{1}{2}$ times the slope of the given line and whose y-intercept is two more than the y-intercept of the given line. Compare the graphs of the two lines.

$$x - 2y = 4$$
$$-2y = 4 - x$$
$$y = \frac{4 - x}{-2}$$
$$y = -2 + \frac{1}{2}x$$
$$y = \frac{1}{2}x - 2$$

slope $= \frac{1}{2}$ y-intercept $= -2$

$$y = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)x - 2 + 2$$
$$y = \frac{1}{4}x$$

The resulting line is less steep and is moved up two units. It passes through the origin.