8.3 teacher notes

Objectives

- Write the equation of a line in slope-intercept form when given the slope of the line and a point on the line.
- Write the equation of a line in slope-intercept form when given two points on the line.

Vocabulary

Parallel lines (Lesson 8-1)
Perpendicular lines (Lesson 8-1)
Point-slope form
Slope-intercept form (Lesson 7-2)
Slope (Lesson 7-2)
Negative reciprocals (Lesson 8-1)

Prerequisites

Finding the slope of a line, given two points on the line

Solving an equation of two variables for a given variable

Finding the slopes of parallel and perpendicular lines

Writing the equation of a line, given the slope and the *y*-intercept

Get Started

- What are two items of information you need to find the equation of a line? the slope and the *y*-intercept
- Given two points on a line, how do you find the slope? Use the slope formula, $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.
- Find the equation of a line in slope-intercept form that has a slope of 2 and a *y*-intercept of 5. $y = 2x + 5$

Section 1

Expand Their Horizons

In Lesson 2, students wrote the equations of lines in slope-intercept form when given the slope and y-intercept. In this lesson, this concept will be extended to include writing the equation of a line given a point and the slope and writing the equation of a line given two points.

Students at this point are often intimidated by lengthy processes. As the lesson progresses, pause frequently and point out steps with which the students are familiar. Students have learned to find slope. They have learned to write equations in slope-intercept form. They have learned to solve equations for a given variable. Point out these steps to help ease some of the intimidation that some students will be feeling at this point.

In the equation $y = mx + b$, the variables, x and y, are used to represent any point on the line. When subscripts are used such as in the slope formula, $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$, the variables represent specific points. In the point-slope form, $y - y_1 = m(x - x_1)$, the variables without subscripts, x and y, represent any point on the line. The variables with subscripts, x_1 and y_1, represent specific points.

The first example involves finding an equation to fit the skunk sales and continues the process of understanding. Roxy states that the increase $10,000 per year must be the slope of the line. How does she know this? The sales increase $10,000 each year. Therefore, if the sales were plotted on a graph, any one year to the next would go up $10,000 and to the right one year. If students do not understand this concept, list some year and sales ordered pairs: (1, 10,000), (2, 20,000), (3, 30,000) etc. Use these pairs to find the slope.

In the second example, students find the equation of the line through the point (2, 4) with $m = 3$. In the point-slope form, the variables containing subscripts represent a specific point. Replace x_1 with 2, and y_1 with 4. The point-slope form of the equation is $y - 4 = 3(x - 2)$.

Equations are usually not left in point-slope form. Slope-intercept form is more convenient for graphing. Help students see that writing the equation in slope-intercept form is the same as solving the equation for y.

The third example gives opportunity for reinforcing operations with fractions.

1 Substitute 6 for x_1 and 4 for y_1 in the point-slope equation. Remind students to use the Distributive Property. When you add 4 to both sides of the equation, you get the slope-intercept form $y = \frac{2}{3}x$.

2 When zero is distributed, then the right side of the equation becomes zero. $y = 9$ is a horizontal line. Some students may notice the zero slope and realize that the line is horizontal as soon as the problem is presented.

Some students may not be satisfied with Ms. Fleigle's answer about what to do if the slope is undefined. Help them to see that the slope is undefined when a line is vertical. Choose two points with the same x-coordinate, such as (3, 1) and (3, 5). Draw the vertical line through the points and attempt to find the slope: $\frac{5-1}{3-3} = \frac{4}{0}$, which is undefined.

Common Error Alert

Students will sometimes substitute the coordinates of a given point for x and y instead of x_1 and y_1. Remind them that x and y represent any point on the line and will remain variables in the final equation. The slope-intercept form of the equation does not have subscripts.

Students may use one of the coordinate values for m instead of first finding the slope. Keep stressing that in order to write the equation of a line, you must know the slope. If the slope is not given, use the slope formula to find it.

In the next example, students find the equation of the line through the point (4, 1) that is parallel to the graph of $y = 2x + 3$. Remind students that parallel lines have the same slope. Once students see that the slope of the line that they are trying to find is 2, reword the question for them to read, "Find the equation of the line through the point (4, 1) with a slope of 2." Now they can write the equation in point-slope form.

In the next example, students find the equation of the line through the point (4, 1) that is perpendicular to the graph of $y = 2x - 3$. It is important to continue stressing that parallel lines have the same slope and perpendicular lines have slopes that are negative reciprocals. The slope of the line whose equation they are trying to find is $-\frac{1}{2}$.

3 To find the equation of a line parallel to the graph of $y = \frac{2}{3}x + 7$, use the slope $\frac{2}{3}$. Use the point-slope form. The given point is (0, 0) and the slope is $\frac{2}{3}$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \tfrac{2}{3}(x - 0)$$
$$y = \tfrac{2}{3}x$$

4 The following formula (process or algorithm) shows how to use the slope-intercept form to find the equation of the line. (This is an alternate method to the one used in the lesson.)

Find b by replacing the variables in the slope-intercept equation with the given values.

$$y = mx + b$$
$$3 = \tfrac{1}{5}(-2) + b$$
$$3 = -\tfrac{2}{5} + b$$
$$3\tfrac{2}{5} = b$$

Rewrite the equation in slope-intercept form substituting $\frac{1}{5}$ for m and $3\frac{2}{5}$ for b.

$$y = \tfrac{1}{5}x + 3\tfrac{2}{5}$$

Look Beyond

In more advanced courses, students may find equations of functions that are not linear given points on the graph of the equation. For example, students may be given points such as (0, –3), (3, 6) and (–2, 1) and asked to find the equation of the parabola that passes through those 3 points ($y = x^2 - 3$).

Additional Examples

1. Find the equation in slope-intercept form of the line that contains the point (3, –5) and has a slope of $\frac{1}{3}$.

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = \tfrac{1}{3}(x - 3)$$
$$y + 5 = \tfrac{1}{3}x - 1$$
$$y = \tfrac{1}{3}x - 6$$

2. Find the equation in slope-intercept form of the line that contains the point (–1, 2) and is parallel to the graph of $y = \frac{1}{4}x - 3$.

$$m = \tfrac{1}{4}$$
$$y - y_1 = m(x - x_1)$$
$$y - 2 = \tfrac{1}{4}(x - (-1))$$
$$y - 2 = \tfrac{1}{4}(x + 1)$$
$$y - 2 = \tfrac{1}{4}x + \tfrac{1}{4}$$
$$y = \tfrac{1}{4}x + 2\tfrac{1}{4}$$

Section 2

Expand Their Horizons

In Section 2, students find the equation of a line given two points on the line. Because of the difficulty of the material, you may want to have students do Section 1 and Section 2 on different days.

The process they will follow is very similar to the process used in Section 1. However, when the slope of the line is not known, the slope formula must first be used to find the slope.

Students may question why the points for the skunk sales are (1, 12) and (4, 57) instead of (12, 1) and (57, 4). Usually the independent variable is written on the *x*-axis, and the dependent variable is written on the *y*-axis. In this case the number of sales is dependent on the month. As time passes, more skunks are sold. If skunk sales are on the vertical axis, then this increase in sales can be shown visually.

Because the slope of the line is not given, the slope formula must be used before the equation can be found. The slope of this line is 15. Help students see that this slope shows that skunks are sold at the rate of 15 each month.

Once the slope is found, choose either point, and substitute its coordinates and the slope into the point-slope form. Even after being told to choose either point to substitute into the point-slope form, many students will continue to ask which point to use. The next example emphasizes the fact either point can be used.

In this example, the points (1, 9) and (3, 3) are given. Have students pay especially close attention to the portion of the video that works the problem first using the point (3, 3) and then using the point (1, 9). Stress the fact that the same equation is found regardless of which point is used.

5 Some students will work this problem using (9, 5).

$$y - 5 = 1(x - 9)$$
$$y - 5 = x - 9$$
$$y = x - 4$$

6 This problem is a little more difficult. It may be necessary to help students one step at a time. Parallel lines have the same slope. Therefore, the slope of the line through the given points is the same as the slope of the line the students are trying to find. The line has a slope of $-\frac{3}{5}$. Substitute this slope and the point (2, 0) into the point-slope equation to find the equation of the line. Emphasize that in this problem, only the point (2, 0) can be substituted for (x_1, y_1).

Additional Examples

1. Find the equation of the line in slope-intercept form of the line through the points (3, –1) and (5, –4).

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-4 - (-1)}{5 - 3} = \frac{-3}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{-3}{2}(x - 3)$$
$$y + 1 = \frac{-3}{2}x + \frac{9}{2}$$
$$y = \frac{-3}{2}x + \frac{7}{2}$$

2. Find the equation in slope-intercept form of the line that passes through the point (0, 3) and is perpendicular to the line through the points (–2, 5) and (6, 2).

For the line through (–2, 5) and (6, 2), the slope is $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{2 - 5}{6 - (-2)} = \frac{-3}{8}$. So, use $m = \frac{8}{3}$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{8}{3}(x - 0)$$
$$y - 3 = \frac{8}{3}x$$
$$y = \frac{8}{3}x + 3$$