8.1 teacher notes

Objectives

- Find the slope of a line given the graph of the line.
- Find the slope of a line given two points on the line.
- Find the slope of horizontal and vertical lines.

Prerequisites

Evaluating expressions using formulas

Vocabulary

Formula (Lesson 3-6)
Coordinate plane (Lesson 7-1)
Slope (Lesson 7-2)
Parallel lines
Perpendicular lines
Reciprocal (Lesson 2-3)
Absolute value (Lesson 1-2)
Negative reciprocals

Get Started

- What is the slope-intercept form of a linear equation? $y = mx + b$.
- In the slope-intercept form of a linear equation, what do the parameters m and b represent? m is the slope and b is the y-intercept.
- How would you graph the line $y = \frac{2}{3}x + 1$? Start at the origin. Move up on the y-axis one unit, and plot (0, 1). From that point, move up two units and to the right three units. Plot that point. Draw a line through those two points.

Section 1

Expand Their Horizons

In the previous module, slope was used to graph lines. In this section the slope of a line will be found by inspection using the ratio rise to run. The ratio rise to run will be used to develop the slope formula. That is, the slope of a line through (x_1, y_1) and (x_2, y_2) is equal to $\frac{(y_2 - y_1)}{(x_2 - x_1)}$.

The slope of a line is constant. Therefore, any two points on a line can be used to find its slope.

In the first examples, students find the slope of a given line using the ratio rise to run. Remind them that they are familiar with this procedure. Have them count along with the characters in the video.

In the second example, the slope of the line is 2. Students sometimes become confused when the slope is an integer. Help them see that 2 can be written as the fraction $\frac{2}{1}$, which means rise two units and run one unit.

As students begin finding the slope of lines with negative slope, remind them that up and right are positive directions. Down and left are negative directions. You may need to show them that $\frac{-2}{3}$ is equal to $\frac{2}{-3}$.

Connections

Slope is used extensively in the real-world. It is important to know the slopes of roads, roofs, ski slopes, wheelchair ramps, and lawn mower handles. If the slope is too great, the device can become dangerous or ineffective.

Common Error Alert

Another error to watch for is students who think a negative slope has both a negative rise and a negative run. Remind them that a negative divided by a negative is a positive.

1 Have students use several points to find the slope of this line. Have them move from one point on the line to another point on the line, going both from left to right and from right to left. Any points they use will give them the same slope.

The formula for finding slope is $\frac{(y_2 - y_1)}{(x_2 - x_1)}$ or the $\frac{(\text{change in } y)}{(\text{change in } x)}$. Students may be interested in knowing another way to write this formula using Greek letters. The symbol Δ (Delta) means "the change in." We can write the formula as $\frac{\Delta y}{\Delta x}$.

Common Error Alert

Many times when students are finding the slope using the formula, they will write the run in the numerator and the rise in the denominator. You can help alleviate this problem by stressing the words rise over run.

As students begin using the formula for slope, it may be necessary to label the numbers in the ordered pairs as x_1, y_1, x_2, and y_2. Explain to them that sub stands for subscript.

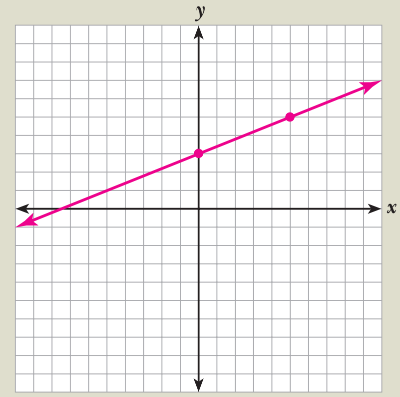
Find the slope of the line through (3, 4) and (5, 2). The most important point to make in this example is that it does not matter which point is (x_1, y_1) and which point is (x_2, y_2). Help students see that $\frac{(2 - 4)}{(5 - 3)}$ is the same as $\frac{(4 - 2)}{(3 - 5)}$. A negative divided by a positive is equal to a positive divided by a negative. $\frac{-2}{2}$ is equal to $\frac{2}{-2}$.

2 For reinforcement, have students plot the points (7, –7) and (–4, 4) and use the ratio rise to run to find the slope.

Additional Examples

1. Use $\frac{\text{rise}}{\text{run}}$ to find the slope of the line.

Students may choose any two points on the graph to count the slope. One example is (0, 3) and (5, 5). Count the rise first, 2. Next, count the run, 5. The slope is $\frac{2}{5}$.



2. Use the slope formula to find the slope of the line through the points (1, 4) and (–3, 7).

Use the slope formula, $m = \frac{(7 - 4)}{(-3 - 1)}$. The slope is $\frac{3}{-4}$ or $\frac{-3}{4}$.

Section 2

Expand Their Horizons

In Section 2, students will find the slopes of horizontal and vertical lines. Before beginning this section, have students simplify the expressions $\frac{0}{1}$ and $\frac{1}{0}$. They should know that $\frac{0}{1}$ is zero and $\frac{1}{0}$ is undefined. This concept will be important as students begin finding the slope of horizontal and vertical lines.

Find the slope of the line through (2, 4) and (6, 4). Many of the students will be like Ferd. They will assume that if the slope is zero, then there is no slope. The video does an excellent job of explaining this concept. Zero is a number. Therefore, the line has a slope.

As students continue their study of mathematics, they will encounter functions. A function is a set of ordered pairs (x, y) in which no two x-coordinates are the same. In a vertical line, all the x-coordinates are the same. A vertical line is the only line that is not a function. A vertical line is also the only line that has an undefined slope.

3 The students have two methods for working this problem. They could substitute the numbers into the slope formula getting $\frac{1}{0}$ as the solution. This is an undefined slope. They could also plot these points. The points form a vertical line when connected. The slope of a vertical line is undefined.

In later lessons students will be finding equations of parallel lines. As they see that parallel lines have the same slope, ask them what is different about the lines. Lead them to the conclusion that parallel lines have the same slope, but different y-intercepts.

For practice, have students find negative reciprocals of several numbers, $\frac{1}{2}$ and –2, $\frac{7}{9}$ and $\frac{-9}{7}$, 3 and $\frac{-1}{3}$.

As the student reviews perpendicular lines, remind them that lines with positive slopes rise to the right. Lines with negative slopes rise to the left or fall to the right.

4 In order to find the slope of a line parallel to a given line, students must first find the slope of the given line. The slope of this line is $\frac{-4}{3}$. Since parallel lines have the same slope, the slope of a line parallel to the given line is also $\frac{-4}{3}$.

Look Beyond

In beginning calculus, students will be finding the slope of lines that are tangent to a curve.

Additional Examples

1. Find the slope of the line passing through the points (–3, 5) and (–3, 8).

Use the slope formula. $\frac{8-5}{-3-(-3)} = \frac{3}{0}$. The slope is undefined.

OR

Plot the points. These points lie in a vertical line. Vertical lines have undefined slopes.

2. Find the slope of a line perpendicular to the line passing through the points (3, 4) and (7, –4).

The slope of the given line is $\frac{-4-4}{7-3} = \frac{-8}{4} = -2$. The slope of the perpendicular line is the opposite reciprocal, $\frac{1}{2}$.