7.4 teacher notes

Objectives

- Write and solve linear equations of two variables to find solutions to business/consumer problems.
- Write and solve inequalities of two variables to find solutions to business/consumer problems.

Vocabulary

Quadrant (Lesson 7-1)
Slope (Lesson 7-2)
Variable (Lesson 3-2)

Prerequisites

Solving linear equations of one variable

Solving linear inequalities of one variable

Graphing linear equations using intercepts

Graphing linear equations using the slope-intercept method

Get Started

- Think of a situation in the real world that could be represented by a linear equation. Answers may vary: repair costs for plumbing, profit, loss, etc.
- How could this situation be changed so that it could be represented by a linear inequality? Change the restrictions to make something a minimum or a maximum, requiring more or less than a quantity.
- Think of jobs whose pay schedule could be represented by a linear equation. A job that pays a set rate plus a commission would be in this category. \$200 a week plus a 5% commission on sales could be represented as $p = 200 + 0.05s$ where p is pay and s is sales.

Section 1

Expand Their Horizons

A linear application can be solved using the four-step method of problem solving. Ferd's problem of figuring out what he had eaten could be solved using this method.

1. **Assign a variable to represent the unknown quantity(ies).**
2. **Write an algebraic equation that states the relationship described in the problem.**
3. **Solve the equation. If solved by graphing, be sure to eliminate any extraneous solutions.**
4. **Write the answer to the problem in a sentence; be sure to use appropriate units.**

Ferd purchased pizza and grease chips from the concession stand. Grease chips cost $2.00 a bag. Pizza costs $4.00 a slice. If Ferd spent $12 in all what possible combinations of grease chips and pizza could he have purchased?

1. **Assign the variables.**
   c = number of bags of grease chips
   p = number of slices of pizza
2. **Write the equation.**
   $\$2.00c + \$4.00p = \$12.00$

If students have trouble understanding this equation, show them these examples: if Ferd bought three bags of chips, he spent $\$2.00 \times 3 = \$6.00$ on chips. If Ferd ate five slices of pizza, he spent $\$4.00 \times 5 = \$20.00$ on pizza.

3. **Solve the equation.**

Because there are two variables and only one equation, there are an infinite number of solutions to this equation. These solutions are represented by the graph of the equation, so there are no *x*'s and *y*'s in the equation. Let c represent the horizontal axis and let p represent the vertical axis. Find the c-intercept by letting p equal zero. The c-intercept is 6. This corresponds to the point (6, 0). Find the p-intercept by letting c equal zero. The p-intercept is 3. This corresponds to the point (0, 3). Graph the line by plotting and then connecting the two intercepts. Remind students of the meanings of these intercepts:

- How many bags of chips did Ferd buy if he bought no pizza? 6
- How many slices of pizza did Ferd buy if he bought no bags of chips? 3

Emphasize that a solution to the equation is not necessarily an answer to the problem. Because the coordinates of the ordered pairs represent the number of slices of pizza or the number of bags of chips that Ferd purchased, any negative coordinate represents a negative number of items. It is not possible to purchase a negative number of items. Continue the discussion to help students understand that Ferd could not have purchased a fractional

Common Error Alert

- Remind students to take time to write out what each variable stands for. Otherwise, they may substitute values incorrectly.
- Typically, the horizontal axis is labeled as x and the vertical axis is labeled as y. When other variables are introduced, students will often plot their points at the wrong coordinates. It may help them to replace the x or y with the assigned variable on the axis it represents.
- Students may see solutions that involve answers such as 2.5 people were present or –3 cookies were eaten. Ask students questions about the problems that help them to see what the problem is talking about. Have them explain the problem in their own words and write what it is that they are trying to find. Have them explain how their solutions make sense within the boundaries of the problem.

portion of an item. By inspection of the line, students should see that the only possible solutions are (6, 0), (0, 3), (2, 2), and (4, 1), the points with whole number coordinates. They can eliminate the first two solutions because Ferd bought both chips and pizza. There is no mathematical way to eliminate the other two. This would be good time to initiate a conversation concerning multiple solutions to a problem. Many students are very uncomfortable with the idea of having more than one correct solution to a problem. Help them to see that both (2, 2) and (4, 1) are possible answers to the problem. The only reason that the characters arrived at a single solution is because Ferd remembered having two bags of chips.

4. **Write the solution in sentence form.** Ferd ate two bags of grease chips and two slices of pizza. Although the video does not explicitly list this form, it does follow this format. Teaching students to use this form will help them to organize their thoughts and their work.

Football Ticket Special. Receive a $0.75 cent discount on your football ticket purchase for every *A* on your report card. Lizzie spent $28.25. The tickets cost $4.00 each. Write an equation to represent this information.

1. **Assign the variables.**
   t = number of tickets.
   a = number of A's.
2. **Write an equation.** The cost of the tickets minus Lizzie's discount is the amount she spent. The cost of the tickets is $4t. The amount of the discount is 0.75*a*. The equation is $\$4.00t - \$0.75a = \$28.25$.
3. **It is not necessary to solve.** The problem only asked for an equation.

1 Ask students, "What information do you have?" Let them arrive at the conclusion that Newt spent $\$1 \times a$ on candied ants and $\$2 \times g$ on grilled grubs, and that he spent $12 altogether. Then ask the students, "Which variable has a value assigned to it?" The students should say "*a*." The problem indicates how many bags of candied ants Newt bought, so students can replace the *a* in the equation with 4.

2 If students are not able to assign variables right away, ask questions such as, "On what two criteria are the raw score based?" "What are we trying to find?" Point out that the number of points gained for *x* correct answers can be found by multiplying $1 \times x$, and the number of points lost for y incorrect answers can be found by multiplying $\frac{1}{4} \times y$. Remind students to check what the variables represent. They should substitute 42 for *x* and solve for *y*.

A DJ charges a one-time set-up fee of $75, then he charges $50 per hour for playing the music. Write an equation to represent the DJ's total charge.

Ask students, "Which variable would be graphed on the horizontal axis and which would be graphed on the vertical axis?" *h, c* Note to students that they could also graph the equation $c = 50h + 75$ and find the *h* coordinate when the *c* coordinate is 300.

Look Beyond

As students continue their mathematics education, applications of this type will become more complicated. They will be graphing several inequalities on the same coordinate plane to solve problems by means of linear programming. At times these equations become very involved in the real world and include as many as 1,000 equations and 1,000 variables. Although the mathematician must possess the skills necessary to set up these problems, these actual computations are done by computer.

Additional Examples

1. Car-go Rentals charges a one-time fee of $50 and $0.35 a mile for a certain car. If Sydney paid $137.50 to rent the car, how many miles did she drive?

1. **Assign the variables.**
   m = number of miles driven
   c = cost to rent the car
2. **Write the equation.** The total cost is the sum of the one-time fee and the product of the miles driven and $0.35. The equation is $c = 50 + 0.35m$.
3. **Solve the equation.** In this case we know that Sydney paid $137.50 to rent the car. Substitute this amount for c in the equation and then solve for m.

$$137.50 = 50 + 0.35m$$
$$137.50 - 50 = 0.35m$$
$$87.50 = 0.35m$$
$$\frac{87.50}{0.35} = m$$
$$250 = m$$

4. **Write the solution as a sentence.** Sydney drove 250 miles.

2. Charles gets paid for baseball at his school. He gets paid $4.00 an hour for practices during the school day and $6.00 an hour for practices after school. One week he made $41 coaching his team. If he had practice five hours during the school day, how many hours did the team practice after school during the week?

1. **Assign the variables.**
   s = hours practiced at school
   a = hours practiced after school
2. **Write the equation.** Charles' pay is equal to the sum of the amount he makes during school and after school. The equation is $4s + 6a = 41$.
3. **Solve the equation.** Charles practiced five hours during school. Substitute 5 for s and then solve for a.

$$4(5) + 6a = 41$$
$$20 + 6a = 41$$
$$6a = 41 \times 20$$
$$6a = 21$$
$$a = \frac{21}{6}$$
$$a = 3.5$$

4. **Write the solution as a sentence.** Charles practiced with the team for 3.5 hours after school.

Section 2

Expand Their Horizons

In Section 2, students are introduced to application problems that involve linear inequalities of two variables. Remind them of words that can be replaced with each of the inequality signs.

>	<
Greater Than	Less Than
More Than	Fewer Than
Over	Under
≥	**≤**
Greater Than or Equal To	Less Than or Equal To
At Least	At Most
No Less Than	No More Than

The admission price for the dance is \$4.50 for boys and \$6.50 for girls. How many people must attend the dance to make at least \$900 in ticket sales?

Connections

Linear equations and inequalities are used extensively in the real world. Mathematicians who are skilled in the process of linear programming are hired by companies to determine the most efficient means of operating businesses. These mathematicians determine trucking routes, fiber optic routes, employee salaries, production schedules, and many other necessary components of business.

Help students pick the sign, greater than or equal to, to represent the words, "at least." $g \geq 94\frac{11}{13}$. This solution presents a problem. There cannot be $\frac{11}{13}$ of a girl attend the dance. Therefore, because Roxy wants at least \$900, the fraction must be rounded up to 95. Even if the fraction were very small, it would still be rounded up to 95.

3 The variables are assigned in the problem, so the first step in problem solving is done. This step is unnecessary. The maximum Newt has to spend is *s* dollars. This amount must be greater than or equal to the membership cost. The membership cost is the sum of the \$75 joining fee and the monthly membership fee of $\$20 \times m$.

4 Notice that the inequality in the video has been turned around to read $20m + 75 \leq 210$. Newt cannot join for a fraction of a month. He only has \$210. Therefore, any decimal part of a month must be rounded down.

5 Although students may assume that this application requires an inequality because it is in the inequality section, point out that the exact amount spent is known, so the application will require an equation $6a + 4c = 24$. Help students realize that the only points that give meaningful answers to the problem are points with whole number coordinates. Points with whole number coordinates lie either in quadrant one or on a positive axis. For this problem, these points are (4, 0), (2, 3), and (0, 6).

Additional Examples

1. Savannah is taking a test. She gets one point for each multiple-choice question she answers correctly and four points for each open-response question she answers correctly. She needs at least 200 points on the test. Write an inequality to model the situation.

1. **Assign the variables.**
   m = number of multiple-choice questions answered correctly.
   r = number of open-response questions answered correctly.
2. **Write an inequality.** $1m + 4r \geq 200$

2. Use the inequality written for question 1 to determine the minimum number of open-response questions Savannah will need to answer correctly if she gets 78 multiple choice questions correct.

1. **Solve the inequality.**
   Replace the m with 78 and rewrite the inequality as $1(78) + 4r \geq 200$.
   $4r \geq 200 - 78$
   $4r \geq 122$
   $r \geq \frac{122}{4}$
   $r \geq 30.5$
2. **Write the solution as a sentence.**
   Savannah must answer at least 31 open-response questions correctly.