7.3 teacher notes

Objective

- Graph linear inequalities of two variables.

Prerequisites

Solving inequalities

Graphing inequalities of one variable on a number line

Graphing linear equations

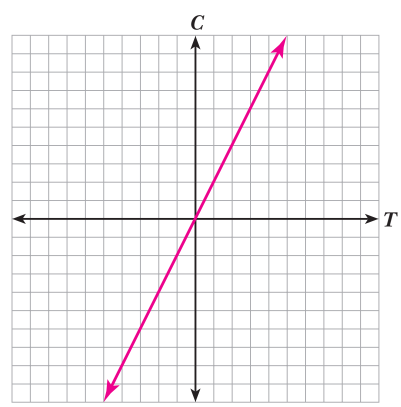
Vocabulary

Boundary line
Coordinate plane (Lesson 7-2)
Inequality (Lesson 5-1)
Number line (Lesson 1-1)
Solution (Lesson 3-2)
Variable (Lesson 3-2)

Get Started

Have students discuss some of the following.

- Write the following statement algebraically, "Cindy makes twice as much money as Thomas"? $C = 2T$
- Graph the equation you just wrote.
- Write the following statement algebraically, "Cindy makes at least twice as much money as Thomas"? $C \geq 2T$



Today we are going to learn to graph inequalities like $C \geq 2T$ on a coordinate plane.

Section 1

Expand Their Horizons

When graphing on a number line, a closed circle will indicate that the value is part of the solution. An open circle will indicate that the value is not part of the solution. One way to remember the correct way to shade on a number line is to shade in the same direction the arrow points, if the variable is on the left.

Graph $x \leq 5$. First have students graph the line $x = 5$. It may be helpful at first to have the students plot several points that satisfy this condition. {(5,0), (5,3), (5,–2), etc}. The students will see that this is the graph of a vertical line.

Students at this level are often still struggling with the meanings of the $<$, $\leq$, $>$, and $\geq$ signs. They may have trouble seeing that $0 < 5$ is a true statement. You may want to show them several examples, reading the sign to them as you go. Ask, "Is zero less than or equal to five? Is negative three less than or equal to five? Is five less than or equal to five?" When asking these questions, write the symbols, $0 \leq 5$, $-3 \leq 5$, $5 \leq 5$. Because five is less than or equal to five, the line $x = 5$ is a part of the graph. Therefore, the line is solid. Shade to the left of the line where $x = 0$ and $x = -3$. Students should realize that the rule for shading in the coordinate plane is similar to the rule for shading on a number line. When the variable is on the left, the graph will be shaded in the direction the inequality sign is pointing.

Graph $y > 2$. Begin by graphing $y = 2$. Once again, plot several points that satisfy this condition. Examples are (–1, 2), (0, 2), (8, 2), (–4, 2). Draw a horizontal line through the point (0, 2). Determine if the line is solid or dashed. Ask, "Is two greater than two?" If they are not sure of their answer, reinforce the point by asking, "If you have two apples, is that more than two apples?" Students should say, "No." Then, tell them that this means that the points on the line $y = 2$ are not solutions. Therefore, it is a dashed line. Although the point (0, 0) is used in the video to determine where to shade, for clarification several other points may be used. Show students that all points above the line make $y > 2$ a true statement. Then shade. Students may notice that the sign is greater than and that the shading is above. This relationship is true for any inequality in the form $y >$ ____, provided that the y appears alone on the left side.

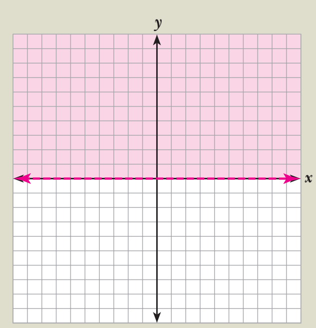
1 Graph $x > -3$. Ask, "Is –3 greater than –3?" Because the answer to this question is no, the boundary line will be dashed. Test several more points to determine which side of the boundary line to shade.

Common Error Alert

Students often try to graph an equation such as $x = 5$ as a horizontal line and an equation such as $y = 5$ as a vertical line because they know that the x-axis is horizontal and the y-axis is vertical. Suggest that students plot several points before actually graphing equations like these.

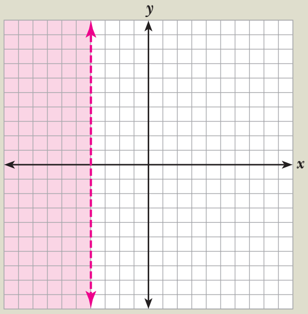
Additional Examples

1. **Graph $y > 0$.** First graph the boundary line $y = 0$. Plot the points (0, 0), (3, 0), and (–2, 0). This boundary line is a special case because it is the x-axis. Because y is greater than 0, the line will be dashed. Deciding how to indicate a dashed line "on top of" an axis that is already solid may require some creative thinking. Next, choose a test point. (0, 0) cannot be used because it lies on the boundary line. Choose (0, 1). $1 > 0$. This is a true statement. Shade above the x-axis.



Additional Examples

2. Graph $x \leq -4$. First, graph the boundary line $x = -4$. Plot the points (–4, 0), (–4, 2), and (–4, –2). This is a vertical line. The line is solid because x is less than or equal to –4. Pick a test point, (0, 0). $0 \leq -4$ is a false statement. Do not shade the region where (0, 0) lies. Shade to the left of the line $x = -4$.



Section 2

Expand Their Horizons

Graph $x + y < 6$. These problems may be a little more difficult for the students to grasp. There are several ways to go about plotting the boundary line $x + y = 6$. Help students see that (0, 6), (6, 0), and (2, 4) are points on the boundary line by writing the equations $0 + 6 = 6$, $6 + 0 = 6$, and $2 + 4 = 6$. Ask if students know any other pairs of numbers that satisfy the condition $x + y = 6$. Suggested pairs might include (–2, 8) and (–4, 10). One way to organize this information is in a chart.

x	y
0	6
6	0
2	4
–2	8
–4	10

Once the line is graphed, have the students check a point either above or below the line to test for shading. (0, 0) is the easiest point to test when it is not on the boundary line, so use it here. Points such as (7, 2), (8, 5), and (–3, 12) that are above the boundary line will make the inequality false.

Graph $y \geq 2x$. Begin by graphing the boundary line $y = 2x$. Create a table of values with points other than those mentioned in the video. Test point, (0, 0) cannot be used because it lies on the boundary line. The characters in the video chose to use (1, 0). Point out that these are small numbers that are easy to use. Because $0 \geq 2$ is a false statement the region below the line containing the point (1, 0) is not shaded. To prove accuracy, test a point above the line such as (2, 5). $5 \geq 2(2)$. $5 \geq 4$. This is a true statement. The region above the line is correctly shaded.

2 Graph $x < 3y$. Remind students that the boundary line for this inequality will be dashed because x must be less than $3y$. It cannot be equal to $3y$. Ask students, "Could (0, 0) have been used as the test point?" The answer to this question is no. The point (0, 0) lies on the line $x = 3y$.

Connections

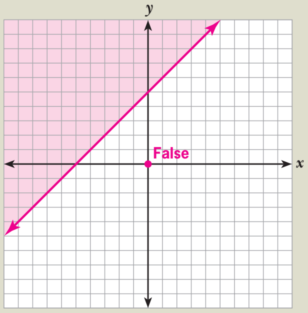
Graphing inequalities has many applications in the real world. One of these applications involves the manufacture of materials used to support structures such as bridges. Manufacturers of these materials, along with the engineers and draftsmen, must understand the concept of linear inequalities and be able to interpret graph of linear inequalities. Inequalities are used to determine if the materials will support enough weight to make the bridge safe and able to sustain rain, wind, and water height and pressure.

Additional Examples

1. $x - y \geq 5$. First graph the boundary line $x - y = 5$. The students may find these values easily. If not, use a table.

x	y
6	1
5	0
0	–5

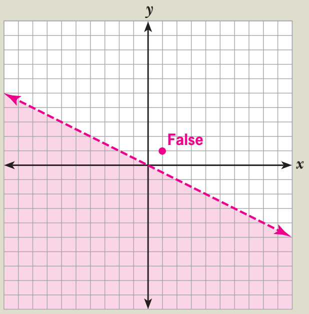
Plot these points. The line is solid because $x - y$ is greater than or equal to 5. Pick a test point (0, 0). $0 - 0 > 5$, so $0 > 5$ is false. Do not shade the region containing (0, 0). Shade below the line.



2. Graph $x < -2y$. Graph $x = -2y$. This time choose values for y and find x. This will be much easier than choosing values for x. Use a table.

y	$-2y$	x
0	$0 = -2(0)$	0
1	$-2 = -2(1)$	–2
–4	$8 = -2(-4)$	8

Plot these points. The line will be dashed because x must be less than $-2y$. Choose a test point. (0, 0) cannot be used because it lies on the line. Choose (1, 1). $1 < -2(1)$ or $1 < -2$ is false. Do not shade the region containing (1, 1). Shade below the line $x = -2y$.



Common Error Alert

Students may not always remember when to use solid and dashed lines. One device could be to tell them that if the symbol has a line underneath it (the part of the equal sign in $\geq$ and $\leq$), the boundary line is part of the solution set, so it is solid.