7.2 teacher notes

Objectives

- Graph linear equations from a data table.
- Graph linear equations using the intercept method.
- Graph linear equations using the slope-intercept method.

Vocabulary

x-coordinate (Lesson 7-1)
y-coordinate (Lesson 7-1)
Ordered pair (Lesson 7-1)
Origin (Lesson 7-1)
Constant (Lesson 1-6)
x-intercept
y-intercept
Slope

Prerequisites

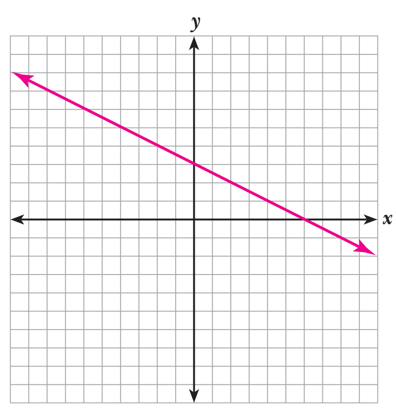
Solving equations

Graphing ordered pairs

Get Started

Have students discuss some of the following.

- Name some points that make $x + 2y = 6$ true. (0, 3), (6, 0), (2, 2), (4, 1), etc.
- Name some points that make $x + 2y = 6$ false. (0, 0), (1, –1), (9, 0), etc.
- How can these questions be answered simply by looking at the graph? All of the points on the line are solutions; any other point is not a solution.



Section 1

Expand Their Horizons

The previous lesson relied mainly on the process of discovery to explore linear equations. This lesson explores these equations more fully and finds new ways to create their graphs.

Graph $3x + y = 12$. Graphing with a table is introduced with this problem. Most students will not have the ability to work the problems in their heads and fill in the table. They will need to work the problems for themselves. Treat corresponding values in the table as ordered pairs. Notice that the video chooses 0, 1, and 2 as values for x. Why are three values chosen? These values are convenient. Although two points determine a line, the third point is needed as a check. Any values could have been used. Although it is customary to choose values for x and solve for y, in some cases it may be easier to choose values for y and solve for x.

Begin by letting x equal zero.
$3(0) + y = 12$
$0 + y = 12$
$y = 12$
This is the ordered pair (0, 12).

Let $x = 1$.
$3(1) + y = 12$
$3 + y = 12$
$y = 12 - 3$
$y = 9$
This is the ordered pair (1, 9).

Let $x = 2$.
$3(2) + y = 12$
$6 + y = 12$
$y = 12 - 6$
$y = 6$
This is the ordered pair (2, 6).

Next, plot these points. Begin at the origin. (0, 12) is the point 0 units over and 12 units up. Therefore, it lies on the y-axis at 12. (1, 9) is the point 1 unit to the right of the origin and 9 units up. This point lies in the first quadrant. (2, 6) is the point 2 units to the right of the origin and 6 units up. This point also lies in the first quadrant.

To prepare students for upcoming material, have them notice that this line rises to the left and falls to the right. This information will help the students get ready to find the slope of a line.

Graph $x - 2y = 8$. Notice that for this problem, the characters in the video do not choose 0, 1, and 2. Stress to the students that any value may be chosen for either variable. This time Newt chooses to let $y = 0$, 1, and –1. Point out to students that this line rises to the right and falls to the left.,

Common Error Alert

- Students sometimes will try to find the x-intercept by setting x equal to zero and try to find the y-intercept by setting y equal to zero. Point out that x coordinate is always zero on the y-axis and y coordinate is always zero on the x-axis.
- The most common error graphing in slope-intercept form occurs after the student has found the y-intercept and is ready to count the slope. Many times students will begin again at the origin instead of at the y-intercept. Help students avoid this mistake by using the slope and counting several points on the line. Stress to them that the y-intercept must be a point on the line.

1 Graph the equation using a table. $4x - 2y = -16$. Although the video uses the values –3, 0, and 1 to replace x, students may vary in their choices. However, all students should have the same line regardless of the values chosen.

Graph $x - 3y = 6$. Notice that the ordered pairs for this equation are found regardless of whether an initial value was chosen for x or for y. The concept of x and y intercepts is introduced. The x-intercept is the point where the line crosses the x-axis. Notice that y always equals zero at the x-axis. The y-intercept is the point where the line crosses the y-axis. Notice that x always equals zero at the y-axis.

Let $y = 0$.
$x - 3(0) = 6$
$x - 0 = 6$
$x = 6$
This is the ordered pair (6, 0).

Let $x = 0$.
$0 - 3y = 6$
$-3y = 6$
$y = \frac{6}{-3}$
$y = -2$
This is the ordered pair (0, –2).

Let $y = 1$.
$x - 3(1) = 6$
$x - 3 = 6$
$x = 6 + 3$
$x = 9$
This is the ordered pair (9, 1).

Because the line crosses the x-axis at (6, 0), the x-intercept is 6. Because the line crosses the y-axis at (0, –2), the y-intercept is –2.

Present these general rules for finding intercepts:

- To find the x-intercept let y equal 0 and solve for x.
- To find the y-intercept let $x = 0$ and solve for y.

Graph $3x + 4y = 12$ using intercepts. The x-intercept is 4. This is the ordered pair (4, 0). The y-intercept is 3. This is the ordered pair (0, 3). The line can be drawn by plotting these two points and drawing the line through them.

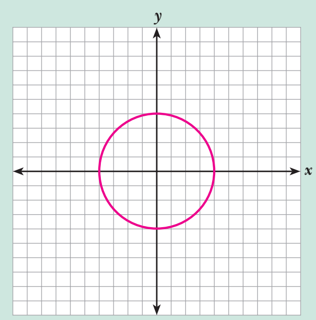
Many students may question this method because they have already been taught to use three points to graph a line. Assure them that two points is sufficient, but that they can always check the line by finding a third point.

2 Graph $2x = y + 6$ using the intercept method. After plotting the intercepts and drawing a line, check this line by choosing a third point.

Look Beyond

Students will use the concepts they learn in graphing linear equations to graph circles, parabolas, and many other shapes as they continue their mathematics education.

Equations such as $x^2 + y^2 = 16$, as shown on the graph, use many similar concepts as graphing by plotting points of graphing using a special form.



Additional Examples

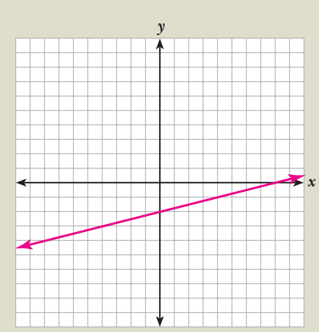
1. Graph the equation $x - 4y = 8$ using a table. Let x equal 0, 2, and 4. The table would look like this.

x	y
0	–2
2	$-1\frac{1}{2}$
4	–1

$0 - 4y = 8$
$-4y = 8$
$y = \frac{8}{-4}$
$y = -2$

$2 - 4y = 8$
$-4y = 8 - 2$
$-4y = 6$
$y = \frac{-3}{2}$ or $-1\frac{1}{2}$

$4 - 4y = 8$
$-4y = 8 - 4$
$-4y = 4$
$y = \frac{4}{-4}$
$y = -1$



Plot these points and connect them with a line to finish.

Additional Examples

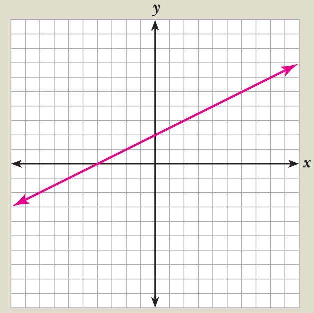
2. Graph the equation $x = 2y - 4$ using the intercept method. The table would look like this.

x	y
0	2
–4	0
–2	1

Let $x = 0$.
$0 = 2y - 4$
$4 = 2y$
$\frac{4}{2} = y$
$2 = y$

Let $y = 0$.
$x = 2(0) - 4$
$x = 0 - 4$
$x = -4$

Let $y = 1$.
$x = 2(1) - 4$.
$x = 2 - 4$
$x = -2$



Plot these points and connect them with a line to finish.

Section 2

Expand Their Horizons

In this section, students will be graphing lines using the slope-intercept method. In other words, they will use the slope of the line and the y-intercept to graph linear equations. Because this lesson contains so much material, consider having students do Section 1 and Section 2 on two different days.

The slope of a line compares the line's rise to its run. The slope is the change of the y-values compared to the change of the x-values. The formula for finding the slope of a line when two points are known is slope $= \frac{y_2 - y_1}{x_2 - x_1}$. A positive slope rises to the right and falls to the left. A negative slope rises to the left and falls to the right. When students are counting to find the slope, stress the importance of using the proper signs. When they count up or to the right, the number is positive. When they count down or to the left, the number is negative.

An equation in the form $y = mx + b$ is in slope-intercept form. In this form, x and y are variables and m and b are numbers. The coefficient of the variable x, which is m, is the slope. The constant at the end of the equation, b, is the y-intercept. The line can be graphed by first plotting the y-intercept and then counting out the slope from the point at the y-intercept. This will give two points on the line. The line can then be drawn connecting these points.

Graph $y = 2x + 6$. This line is graphed first using intercepts. The y-intercept is 6. This is the ordered pair (0, 6). Counting from (–3, 0) to (0, 6), count up six units and to the right three units. The slope is $\frac{6}{3}$ or 2. Help the students discover along with the characters the 2 in the equation. Help them see its position in the equation as the coefficient of the x-term. This coefficient will always be the slope of the line when the equation is written in slope-intercept form. Next, help students find the y-intercept 6. They will notice that there is also a 6 in the equation. This is the constant term. When the line is in slope-intercept form, the constant term will always be the y-intercept of the line.

Graph $y = \frac{3}{2x} - 1$. First find the y-intercept. If some students are having trouble seeing that negative one is the y-intercept, it may help to rewrite that problem as $y = \frac{3}{2}x + (-1)$. Remind students that subtracting a value is the same thing as adding its opposite. Once students have found the y-intercept, they will need to find the slope. The slope is the coefficient of x, $\frac{3}{2}$. Have students plot the y-intercept –1 on the graph and count the slope with the characters on the video. They will count up two units and to the right two units from the point (0, –1) to find another point on the graph. Remind students that a negative divided by a negative is a positive, which means $\frac{-3}{-2}$ is equal to $\frac{3}{2}$. For reinforcement have students count out several points on both ends of the line using both $\frac{3}{2}$ and $\frac{-3}{-2}$ as the slope.

Graph $y = -3x + 2$. First, find the slope –3. Slope is rise over run, slope should be written as a fraction. Negative three is equal to negative three over one, or three over negative one. $\frac{-3}{1} = \frac{3}{-1} = -3$. The constant in the equation is 2. This is the y-intercept. Students should plot the y-intercept. From this point, they use the slope to find the second point. If they use $\frac{-3}{1}$, then they will count down three and over one to the right to end up at the point (1, –1). If they use $\frac{3}{-1}$, then they will count up three and over one to the left to end up at the point (–1, 5). These points are linear and represent the line $y = -3x + 2$.

Graph $y = \frac{x}{2}$. Students may need to see this problem written out first as $y = \frac{1}{2}x + 0$. This may now be a good time to remind students of the identity element for addition, which is zero. $\frac{1}{2}x$ is the same as $\frac{1}{2}x + 0$ because zero is the identity element. Now the equation is in slope-intercept form. $y = \frac{1}{2}x + 0$. The slope is $\frac{1}{2}$. The y-intercept is zero.

3 Begin by rewriting the equation as $y = \frac{-1}{3}x + 0$. In this form students quickly see that the slope is negative one-third and the y-intercept is zero.

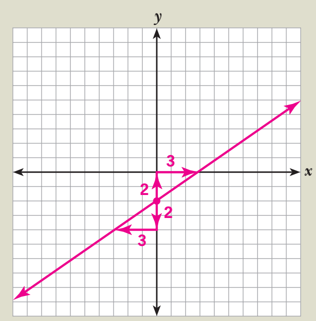
Connections

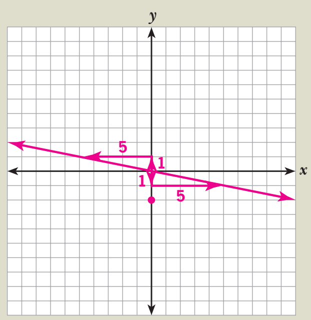
Engineers at road construction sites must be familiar with slope. They grade each hill to find its slope and to ensure the safety of the motorists on the road.

Slopes are also important in building codes. Slopes for handicap accessible ramps should have a slope, or grade, between 1:20 and 1:12 depending on building codes.

Additional Examples

1. **Graph the equation using the slope-intercept method. $y = \frac{2}{3}x - 2$.** The slope is the coefficient of x, $\frac{2}{3}$. The y-intercept is the constant, –2. Begin by plotting (0, –2). Then count up two units and to the right three units to land at the point (3, 0). Check by beginning at the point (0, –2). Count down two units and to the left three units to land at the point (–3, –4). These points are linear. Finish by connecting them with a line.





2. **Graph the equation using the slope-intercept method. $y = \frac{-x}{5}$.** This equation does not appear to be in slope-intercept form. Rewrite the equation as $y = \frac{-1}{5}x + 0$. The slope of this line is $\frac{-1}{5}$. The y-intercept is 0. Graph the line by beginning at the origin. Then, count down one unit and count five units to the right to plot the point (5, –1). Next, begin at the origin and use the slope in the form $\frac{1}{-5}$. Count one unit up and count five units to the left to plot the point (–5, 1). These points are linear. Finish by connecting them with a line.