7.1 teacher notes

Objectives

- Find solutions to linear equations of two variables and write them as ordered pairs.
- Graph points on the Cartesian Coordinate System.
- Graph horizontal and vertical lines from equations.

Vocabulary

Ordered pair
Coordinate
Linear equation
Solution (Lesson 3-2)
Cartesian Coordinate System
Axis (Axes)
x-axis
y-axis
Origin
Coordinate plane
Quadrant

Prerequisites

Solving one-variable equations

Evaluating expressions

Graphing on a number line

Get Started

Have the students discuss some of the following.

- So far we have solved equations that have one variable. How can we solve an equation with two variables? Choose a value for one variable and find the matching value of the other variable.
- Suppose Josh earns $5.50 per hour, and he wants to compare the number of hours he works in a week to the amount of pay he receives. He could have an equation such as $p = 5.5h$, where p is equal to pay and h is equal to the number of hours he works.
- Could he find out how much money he would make if he knew the number of hours he worked? Yes, he could substitute a number for h and simplify to find a value for p.
- Could he find out how many hours to work to make a given amount of money? Yes, he can substitute a value for p and divide by 5.5 to solve for h.

Section 1

Expand Their Horizons

Each variable in a linear equation is of the first degree. In other words, each variable has an exponent of one. The module and lesson both have "two variables" in their names, but we include linear equations of one variable as special cases because they can be graphed on a coordinate plane just as two variable equations are. Some textbooks define a linear equation of two variables to be any equation of the form $ax + by = c$, where either a or b can be zero, but not both.

In the first example, $x + 2y = 6$, remind students that not only is the exponent one understood, but the x term also has an understood coefficient of one.

It is important that students understand that a linear equation of two variables has an infinite number of solutions. In a linear equation, the values of the variables for each solution are dependent on one another.

Be sure when students write an ordered pair, they put the value for x first and put the value for y second. Ordered pairs are always written with the x coordinate first and the y coordinate second.

Note that Roxy, Newt, Lizzie, and Ferd choose an arbitrary number for either x or y. Stress to students that it does not make any difference what numbers they choose, or which variable they choose to replace. The only requirement is that they must solve the equation based on the choices they make.

Common Error Alert

Many times when students are plotting ordered pairs (x, y), they will go up or down first and then left or right. Point out to students that the variables in the parentheses are in alphabetical order, so x is the first coordinate. Because the horizontal line in a standard coordinate plane is the x-axis and runs left to right, go left or right first. Then go up or down.

1. Even though we knew the value of y before we knew the value of x, the value of x is always written first in the ordered pair.
2. Remind students of the steps needed to solve multi-step equations; Lessons 3.4 and 3.5 may be used for review.
3. Encourage students to complete each step to avoid making common computational errors.
4. Many times students will think that they have missed the problem if they get a fraction or mixed number as the solution. Explain to them, that in the real world, solutions are often fractions or mixed numbers. It is acceptable to get a fraction or mixed number as the solution.

Additional Examples

1. **Find the solution of $2x - y = 0$ when y is equal to $\frac{1}{4}$.** Replace y with $\frac{1}{4}$. $2x - \frac{1}{4} = 0$. Add $\frac{1}{4}$ to both sides of the equation. $2x = \frac{1}{4}$. Divide both sides of the equation by 2 to get the solution, $x = \frac{1}{8}$. The ordered pair solution is $\left(\frac{1}{8}, \frac{1}{4}\right)$.

2. **Find the solution of $x = \frac{y}{2}$ when $x = 3$.** Replace x with three. $3 = \frac{y}{2}$. Multiply both sides of the equation by two to get $6 = y$. The ordered pair solution is (3, 6).

Section 2

Expand Their Horizons

In Section 2, students will actually begin to graph the solutions to linear equations. Help them as they begin plotting their solutions to discover that the graphs of the solution sets of all the equations are lines.

For more information on René Descartes see the Look Beyond.

On the coordinate plane, the positive *x* values are to the right of the origin. The negative *x* values are to the left of the origin. The positive *y* values are above the origin. The negative *y* values are below the origin. The quadrants are usually numbered using Roman numerals.

When plotting ordered pairs, **always** begin at the origin. Help students to recognize that the point (4, 1) is in the first quadrant. It may help some students who are more kinesthetic to actually trace the path indicated by an ordered pair. In this case they would trace to the right four units with their fingers and up one unit.

The next point (6, 0) lies on the *x*-axis. The points on any axis and the origin do not lie in any quadrant.

5 The point (0, 3) lies on the *y*-axis in line with the other points.

6 Some students will have a problem graphing this point. Explain to them that they will have to estimate $3\frac{1}{2}$. Students may not be exactly halfway between the 3 and the 4, but they can come close enough.

At this point it should be obvious to the students that the points form a line. After drawing the line, help students pick out some more points on the line and show them that these points also make the equation, $x + 2y = 6$, true. For example, choose the point (2, 2). $2 + 2\ (2) = 6$.

Stress to the students that any point on the line will make the equation, $x + 2y = 6$, true. Also, emphasize to them that any point not on the line will make the equation, $x + 2y = 6$, false. *The solutions to a linear equation **always** lie on a line.*

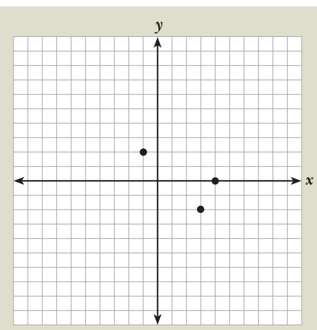
To find all the solutions to a linear equation, find some of the solutions and draw the line through them. Although only two points are necessary, students should plot at least three. Plotting three points will often help students catch any mistakes they may make.

Look Beyond

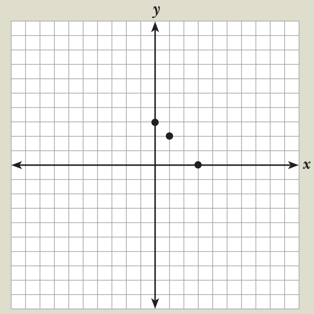
René Descartes was one of the most influential mathematicians of all time. He was also a French philosopher and a scientist. He is often called the founder of analytical geometry because his use of the coordinate plane allowed for the integration of algebra and geometry. He made many contributions to algebra, such as the treatment of negative roots and the convention of exponent notation. He also studied the reflection and refraction of light, wrote a text on physiology, and worked in psychology. One of his best known philosophical statements is "I think; therefore, I am."

Additional Examples

1. **Plot (4, 0), (3, –2), and (–1, 2) on the same coordinate plane.** Begin with (4, 0). From the origin, trace four units to the right on the *x*-axis and zero units up. The point (4, 0) lies on the *x*-axis. Next plot (3, –2). From the origin, trace three unit to the right and two units down. The point (3, –2) lies in the fourth quadrant. Finally, plot the point (–1, 2). From the origin, trace 1 unit to the left and two units up. The point (–1, 2) lies in the second quadrant.



2. Find three solutions to the equation $x + y = 3$ and graph the solution to the equation on the coordinate plane.
Students may begin with any value for x or y. These values will work. Let $x = 0$. Then $0 + y = 3$, or $y = 3$. This is the ordered pair (0, 3). Let $y = 0$. Then $x + 0 = 3$, or $x = 3$. This is the ordered pair (3, 0). Finally, let $x = 1$. Then $1 + y = 3$, or $y = 2$. This is the ordered pair (1, 2). Plot these points and draw the line through them.



Section 3

Expand Their Horizons

In Section 3, students are introduced to horizontal and vertical lines. Discuss some ideas that may help them distinguish between the meanings of the terms horizontal and vertical. Horizontal can be associated with the horizon. The horizon is flat like a horizontal line. Many students this age are interested in their vertical jumps. Help them associate jumping up with a line that goes up and down.

In the equation $y = 3$, it does not matter what value is chosen for x. Emphasize this fact by writing (, 3) on the board and having a student choose a value for x and then plot the point on a coordinate plane on the board. Repeat this several times with other students. Then, have students plot the points along with the video. They should see the horizontal line forming.

7 When points such as (0, –5), (1, –5), and (–2, –5) are connected, they form the horizontal line $y = –5$.

Solve $x = –2$. Students should see the similarity between this equation and the last set of equations. Have them point to places on the graph where x is equal to –2. Then, have them plot the points (–2, 0), (–2, 1), and (–2, –2) along with the characters on the video. They should see that these points lie on a vertical line.

8 Ask, “Are there any points that make the equation, $x = 7$, true that are not found on this line? Are there any points on this line that do not make the equation, $x = 7$, true?” The answer to both of these questions is no.

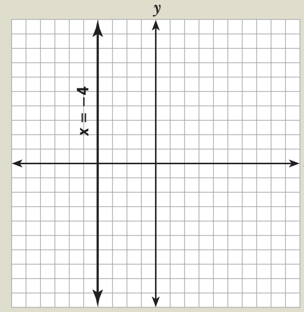
Some students may become confused with the variables in $y = b$ and $x = a$. Explain that an equation in the form y equals a constant (or number) is a horizontal line. An equation in the form x equals a constant (or number) is a vertical line.

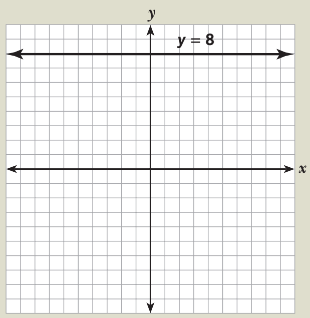
Connections

Linear equations are used extensively in the real world. They can be used to graph expected income or expected weight loss and weight gain. In physics, linear graphs show distance traveled with respect to time when traveling at a constant rate.

Additional Examples

1. **Graph all the solutions to the equation *x* = –4.** Ask, "Is this a horizontal line or a vertical line?" Help students answer this question by plotting points such as (–4, 0), (–4, 3), and (–4, –3). Connecting these points forms a vertical line.





2. **Graph all the solutions to the equation *y* = 8.** Ask, "Is this a horizontal line or a vertical line?" Help students answer this question by plotting points such as (0, 8), (–3, 8), and (2, 8). Connecting these points forms a horizontal line.