6.5 teacher notes

Objective

- Model scenarios using an absolute value equation or inequality, then solve.

Prerequisites

Writing and solving absolute value inequalities using "less than"

Writing and solving absolute value inequalities using "greater than"

Writing and solving absolute value equations

Vocabulary

Absolute value (Lesson 6-1)
Absolute value inequality (Lesson 6-3)
Conjunction inequality (Lesson 5-5)
Disjunction inequality (Lesson 5-6)
Range
Error
Solution (Lesson 3-2)
Maximum
Minimum
Isolate (Lesson 3-1)

Get Started

- Break the class into pairs to play a number line guessing game.
- Have each player first draw a number line that is 11 units long. Say, "Your number line can contain any 11 integers. For example, it might go from 0 to 10, or from –15 to –5, or from 32 to 42."
- Then have each player "hide" a segment that is 2, 4, 6, or 8 units long somewhere along their number line and mark the midpoint of their segment as *m*.
- Say, "You have just created a target range for your opponent. Your opponent's job is to determine the maximum and minimum values of the target range."
- Each player then takes a turn by guessing numbers for their opponents target range. If the number guessed lies outside the range, the opponent says, "Outside the range." If it is an endpoint of the range, the opponent says, "Endpoint." If it is inside the range, the opponent says, "Inside the range."
- Continue until each opponent has determined both endpoints of the target range. Have each pair work together to write an algebraic expression describing the set of points that are inside the range, outside the range, and endpoints of the range.

Section 1

Expand Their Horizons

In this lesson, students will write absolute value equations and inequalities to solve problems.

Before beginning the lesson, remind students of this idea: the distance from a to b on a number line can be described by the expression $|a - b|$. To help students understand this idea, draw a number line on the board, placing points at 5 and 8. Ask, "What is the distance from 5 to 8?" The answer is 3 units. Then ask, What is the distance from 8 to 5?". Again, the answer is 3 units. Write the expressions $|5 - 8|$ and $|8 - 5|$ on the board and have the class simplify each one. Both expressions are equal to three.

Remind the class that absolute value is used to describe distance without regard to direction. This is what makes absolute value equations and inequalities ideal for describing situations in which a "target range" exists. In the first example in the lesson, the boys know only that their water balloon lands "at most 20 feet from the target." Point out that the boys do *not* know whether a particular shot will fall short of the target or pass it. They only know it will miss the target by at most 20 feet. In other words, they only know the distance, not the direction.

When the problem is modeled on a number line, the distance from the actual landing spot to 100 can be expressed as $|d - 100|$. Some students may ask whether the expression $|100 - d|$ is also acceptable. Remind students that they have seen that $|5 - 8| = |8 - 5|$ (or more generally, $|a - b| = |b - a|$), so the expressions $|d - 100|$ and $|100 - d|$ have the same value and are therefore interchangeable. You may want to have students solve the problem using $|100 - d|$ instead of $|d - 100|$. They will see that they arrive at the same answer either way, but using $|100 - d|$ requires extra steps.

When translating a word problem into an absolute value equation or inequality, students will need to identify key words in the problem that indicate which of the five possible signs ($<$, $\leq$, $>$, $\geq$, or $=$) should be used. It may be helpful to create and/or post a list of key words for each sign.

When the equation or inequality reads	The distance from x to a is ($<$, $\leq$, $>$, $\geq$, or $=$) b
• $|x - a| < b$	less than, within (b units of a)
• $|x - a| \leq b$	less than or equal to, no more than, at most
• $|x - a| > b$	greater than
• $|x - a| \geq b$	greater than or equal to, no less than, at least
• $|x - a| = b$	exactly (use to find the maximum and minimum values of the target range)

1 Use the inequality $|x - a| < b$ to represent the idea, "the distance from x to a is less than b units." x is the range of possible values, a is the target value, and b is the width of the acceptable values on either side of a. Use the inequality $|t - 55| < 10$ to show that the distances from the possible temperatures, t, to 55 is less than 10 units. You may want to draw a thermometer to model this problem.

2 This word problem asks for the maximum and minimum possible values for a safe tire. The maximum and minimum values of the range of safe tires can be found by finding the two solutions to the equation $|p - 36| = 3$.

3 In this problem, the alarm will sound when the distance from the actual water level, x, to 20.5 is more than 0.3. The verb phrase "is more than" translates to the inequality symbol $>$. This means students have to solve a disjunction.

Connections

Tell students that it is important to be able to model situations with expressions, equations, and inequalities. Such modeling allows computer programmers to input rules and formulas and to program consequences that are dependent on the results of operating on those formulas. It also allows engineers, architects, and building inspectors to make decisions on the construction and approval of design plans for structures based on safety tolerances.

Additional Examples

Solve and graph.

1. Meadowbrook school was built for an ideal population of 325 students. In any year during which the actual population varies by more than 35 students from the ideal, the principal must file a special report to the school board. For what student populations must the principal file the report?

Identity key information.
Ideal population: 325
Allowable variation: 35
Comparison: by more than ($>$)
Actual population: p

$|p - 325| > 35$

$p - 325 > 35$ or $p - 325 < -35$

$\underline{+325 \quad +325}$ $\quad$ $\underline{+325 \quad +325}$

$p > 360$ or $p < 290$

The principal must file a report when the actual population is more than 360 students or less than 290 students.

2. A certain survey showed that 4,800 voters plan to vote for McPeterson. If the survey results differ from the actual value by at most 300 voters, what are the possible values for the actual number of voters planning to vote for McPeterson?

Identity key information.
Voters in survey: 4,800
Difference: 300
Comparison: at most ($\leq$)
Actual voters: a

$|a - 4{,}800| \leq 300$

$-300 \leq a - 4{,}800 \leq 300$

$+4{,}800 \quad +4{,}800 \quad +4{,}800$

$4{,}500 \leq a \leq 5{,}100$

The actual number of voters planning to vote for McPeterson may be as low as 4,500 or as high as 5,100.