6.4 teacher notes

Objective

- Model scenarios using an absolute value equation or inequality, then solve.

Vocabulary

Absolute value (Lesson 6-1)
Conjunction inequality (Lesson 5-5)
And (Lesson 5-5)
Disjunction inequality (Lesson 5-6)
Or (Lesson 5-6)
Compound inequality (Lesson 5-5)
Isolate (Lesson 3-1)

Prerequisites

Writing disjunction inequalities

Solving disjunction inequalities

Graphing the solution sets to disjunction inequalities on a number line

Get Started

- Break the class into small groups of three or four students each.
- Ask each group to draw a number line from –10 to +10, then to graph all the points that are more than 5 units away from 0. Graphs should consist of two rays: one with an open circle at –5, extending infinitely to the left, and another with an open circle at 5, extending infinitely to the right.
- Have the groups discuss the graph. Ask, "Can your group think of two different ways to express the set of numbers included in the graph?" (If students need to be pointed in the right direction, it may help to remind students that in the previous lesson, they learned that the statements $|x| < a$ and $-a < x < a$ are equivalent.) The graph can be described by the absolute value inequality $|x| > 5$ or by the disjunction inequality $x < -5$ or $x > 5$.

Section 1

Expand Their Horizons

In Section 1, students use the skill of solving disjunction inequalities to solve absolute value inequalities using "greater than." Each inequality in this lesson can be reduced to the form $|\textit{linear expression}| > a$ or $|\textit{linear expression}| \geq a$ (where a is a real number) by using inverse operations. When an inequality takes the form $|\textit{expression}| > a$, the absolute value operation is "undone" by writing the equivalent disjunction $\textit{expression} > a$ or $\textit{expression} < -a$.

Help students see why the inequality $|x| > a$ is equivalent to the disjunction $x > a$ or $x < -a$. The inequality $|x| > a$ means "any number x such that x is more than a units away from 0." On a number line, the numbers that are more than a units away from 0 are those numbers to the right of a and those numbers to the left of $-a$. So, the solutions to the inequality $|x| > a$ can be expressed by the disjunction $x > a$ or $x < -a$.

Review the meaning of *disjunction* with students. A disjunction is true when either or both of its inequalities or true. When the inequality $|x| > 2$ is solved by writing the disjunction $x > 2$ or $x < -2$, it is true for any x such that x is either greater than two or less than –2. Remind students to be careful with the words "and" and "or." A disjunction always uses the word "or."

To solve a disjunction, solve each inequality, one at a time. The solutions to the inequalities of the disjunction are the solutions to the "greater than" absolute value inequality. For example, the solutions to the "greater than" inequality $|x + 6| > 2$ are found by writing and solving the inequalities of the disjunction $x + 6 > 2$ or $x + 6 < -2$.

When an inequality takes the form $|\textit{linear expression}| > a$, students should take a moment to study the inequality before writing the disjunction inequality. If a is positive, they can proceed by writing the disjunction $\textit{linear expression} > a$ or $\textit{linear expression} < -a$. If a is a negative number, then the solution to the "greater than" inequality is all real numbers, since an absolute value expression is *always* greater than a negative number. If a equals zero, the "greater than" inequality is true for all real numbers *except* the value(s) that make the expression equal to 0.

1 The solution to the absolute value inequality can be expressed by the equivalent disjunction $n \geq 3$ or $n \leq -3$. The graph consists of two rays. One has its endpoint at 3 and extends infinitely to the right; the other has its endpoint at –3 and extends infinitely to the left.

2 Students may incorrectly write the disjunction as $9p > 27$ or $9p > -27$. Remind them to think of absolute value inequalities visually, as distance on a number line. The expression $9p$ must have a distance from 0 that is greater than 27, so $9p$ must lie either to the right of 27 (giving $9p > 27$) or to the left of –27 (giving $9p < -27$).

3 Begin by rewriting the absolute value inequality so that the absolute value expression is on the left side. Remind students that the inequalities $-6 < |k + 4|$ and $|k + 4| > -6$ are equivalent (the latter is formed by reading the former from right to left). Because the inequality takes the form $|\textit{expression}| > a$, where a is negative, the solution is all real numbers. No matter what value k takes, the absolute value of the expression $k + 4$ will always be greater than zero, and therefore always greater than –6.

Encourage students to spend a moment studying the inequality before writing the disjunction. If necessary, post a chart showing the steps for solving each type of "greater than" inequality.

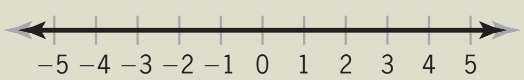
- |*linear expression*| > *negative number*: solution is all real numbers
- |*linear expression*| > 0: solution is all real numbers except the solution to the equation *linear expression* = 0
- |*linear expression*| ≥ 0; solution is all real numbers
- |*linear expression*| > *positive number*: two solutions (solve the disjunction)

4 Since this inequality takes the form |*linear expression*| > 0, it is true for all real numbers except the one that makes the expression equal to zero. The solution is $3d - 4 \neq 0$. To solve, add four to both sides to get $3d \neq 4$, then divide both sides by three to get $d \neq \frac{4}{3}$.

Additional Examples

1. $|p + 6| \geq -4$

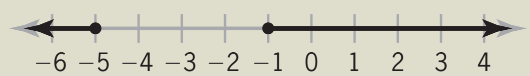
Any value substituted for *p* will result in an absolute value greater than –4.
Solution: all real numbers or $\Re$



2. $|5r + 15| \geq 10$

Write the disjunction and remember to reverse the sign of the inequality for –10 and then solve.

$$5r + 15 \geq 10 \quad \text{or} \quad 5r + 15 \leq -10$$
$$\underline{-15 \;\; -15} \qquad\qquad \underline{-15 \;\; -15}$$
$$5r \geq -5 \quad \text{or} \quad 5r \leq -25$$
$$\frac{5r}{5} \geq \frac{-5}{5} \quad \text{or} \quad \frac{5r}{5} \leq \frac{-25}{5}$$
$$r \geq -1 \quad \text{or} \quad r \leq -5$$



Section 2

Expand Their Horizons

Before solving the equations in this section, review the idea of *isolating* the absolute value expression on one side. Students are already familiar with this process from their experience with solving absolute value equations and "greater than" inequalities. To isolate the absolute value expression, first isolate the absolute value term by adding or subtracting. Next, eliminate any coefficient on the absolute value expression by multiplying or dividing.

As when solving "less than" absolute value inequalities, the absolute value expression **must** be isolated before students can speculate about the number of solutions or write the disjunction inequality.

5 To solve, isolate the absolute value term by subtracting 10 from both sides, then dividing both sides by 2 to get $|y| > -2$. Since this inequality takes the form |*linear expression*| > *negative number*, the solution is all real numbers. The graph of *all real numbers* shows the entire number line shaded.

Common Error Alert

Students may fail to isolate the absolute value expression before writing the disjunction. Remind them that the absolute value expression must be isolated on one side before writing the disjunction inequality.

Additional Examples

Solve and graph.

3. $6|y| - 3 > 15$

Isolate the absolute value expression by first adding three to both sides and then divide both sides by six.

$$6|y| - 3 > 15$$
$$\underline{+3} \quad \underline{+3}$$
$$6|y| > 18$$
$$\frac{6|y|}{6} > \frac{18}{6}$$
$$|y| > 3$$

$y > 3$ or $y < -3$

4. $12 + 2|y - 2| > 0$

First subtract 12 from both sides and then divide both sides by two.

$$12 + 2|y - 2| > 0$$
$$\underline{-12} \quad \underline{-12}$$
$$2|y - 2| > -12$$
$$\frac{2|y - 2|}{2} > \frac{-12}{2}$$
$$|y - 2| > -6$$

Solution: all real numbers or $\Re$