6.3 teacher notes

Objective

- Solve inequalities containing the absolute value expression “is less than” or the absolute value expression “is less than or equal to.”

Vocabulary

Absolute value (Lesson 6-1)
Conjunction (Lesson 5-5)
Compound inequality (Lesson 5-5)

Prerequisites

Writing conjunction inequalities in the form $a < x < b$, where $a < b$

Solving conjunction inequalities

Graphing the solution sets to conjunction inequalities on a number line

Solving absolute value equations

Get Started

- Have students form pairs. Ask each pair to draw a number line, and graph all the numbers between 3 and –3. Remind students that such numbers include not only integers, but rational numbers (fractions and decimals). Graphs should have open circles at –3 and 3, with shading between.
- Ask each pair to write a number sentence to describe the set of numbers between 3 and –3 and a number sentence to describe $-3 < x < 3$.
- Ask each pair to draw another number line and to graph any number whose absolute value is less than 3. Write on the board “Graph every number x so that $|x| < 3$.” Graphs should have open circles at –3 and 3, with shading between.
- Then, ask students to compare their two graphs. The graphs of $|x| < 3$ and $-3 < x < 3$ are identical.

Section 1

Expand Their Horizons

In Section 1, students use the skill of solving conjunction inequalities to solve absolute value inequalities using "less than." Each inequality in this lesson reduces to the form $|\textit{linear expression}| < a$ or $|\textit{linear expression}| \leq a$, where a is a nonnegative real number.

The inequalities $|x| < a$, where a is positive, and $-a < x < a$ are equivalent statements. This idea is essential to understanding the steps taken in solving a "less than" inequality. Students are used to the idea of "undoing" operations to isolate x in an equation or inequality. Writing the conjunction inequality $-a < x < a$ is the way to "undo" the absolute value around the x in a "less than" inequality.

Help students to recognize that the inequalities $|x| < a$, where a is positive, and $-a < x < a$ are equivalent. Write the inequalities $|x| < 7$ and $-7 < x < 7$ on the board. Read the first inequality as "the distance from 0 to a number x is less than 7 units." Show the class how the statement describes the numbers from –7 to 7, not inclusive of the endpoints. If necessary, move up the number line, one integer at a time, asking "Is the absolute value of –8 less than 7? . . . of –7? . . . of –6?"

When an inequality takes the form $|\textit{linear expression}| < a$, students should take a moment to study the inequality before writing the conjunction inequality. If a is positive, they can proceed by writing $-a < \textit{expression} < a$. If a is negative, the inequality can have no solution, since the inequality shows that an absolute value expression is equal to a negative number (if a is negative, then a is a negative number). If a is zero, then the inequality is true only when the linear expression is equal to zero.

1 To solve, write the equivalent inequality $-4 \leq c \leq 4$. In this case, no further operations are necessary since c is already isolated in the middle of the expression.

2 Remind students that the operations the complete to isolate the variable in a compound inequality must also be performed on the boundary expressions (20 and –20 for this example).

3 If $|k + 8|$ is less than –10, it must be equal to a negative number because a number less than –10 is always negative. Therefore, this inequality has no solution.

Students may write and solve the conjunction inequality $10 < k + 8 < -10$. Encourage students to spend a moment studying the inequality before writing the conjunction. If necessary, post a chart showing the steps for solving each type of "less than" inequality.

- $|\textit{linear expression}| < \textit{negative number}$: no solution
- $|\textit{linear expression}| < 0$: no solution
- $|\textit{linear expression}| \leq 0$; one solution (solve $\textit{linear expression} = 0$)
- $|\textit{linear expression}| < \textit{positive number}$: two solutions

4 Remind students that the direction of the inequality must be reversed when multiplying through by a negative number, and that the inequality can be read from right to left as three is less than or equal to x which is less than or equal to nine.

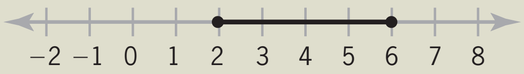
Additional Examples

Solve.

1. $|r - 4| \le 2$

Write the conjunction inequality and add four to all three parts:

$$\begin{aligned} -2 &\le r - 4 \le 2 \\ +4 &\quad\; +4 \quad +4 \\ 2 &\le r \le 6 \end{aligned}$$



2. $\left|\frac{x}{4} - 2\right| \le 0$

The absolute value needs one equation because the absolute value expression is less than or equal to zero. There is only one solution that will have that value.

$$\begin{aligned} \frac{x}{4} - 2 &= 0 \\ +2 \;\; &\; +2 \\ \frac{x}{4} &= 2 \\ (4)\frac{x}{4} &= 2(4) \\ x &= 8 \end{aligned}$$

Section 2

Expand Their Horizons

Before solving the equations in this section, review the idea of *isolating* the absolute value expression on one side. Students are already familiar with this process from their experience with solving absolute value equations. To isolate the absolute value expression, first isolate the absolute value term by adding or subtracting. Next, eliminate any coefficient on the absolute value expression by multiplying or dividing. Point out that in an expression like $\frac{|x|}{2}$, the absolute value expression has a coefficient of $\frac{1}{2}$.

Emphasize that the absolute value expression **must** be isolated before students can speculate about the existence of solutions or write the conjunction inequality.

5 To solve, isolate the absolute value term by subtracting four from both sides to get $4|x| < 16$. Next, divide both sides by four to get $|x| < 4$. Because the inequality now takes the form $|expression| < a$, and $a > 0$, the solution is the conjunction inequality $-4 < x < 4$.

6 To solve, isolate the absolute value expression on the left side by subtracting five from both sides. Since the result, $|7j - 2| \le -2$, takes the form $|expression| \le a$, where $a < 0$, the inequality has no solution.

Common Error Alert

Students may fail to isolate the absolute value expression before writing the inequality. They may write $-3 \le 5 + |7j - 2| \le 3$. Remind them that the absolute value expression must be isolated on one side before writing the conjunction inequality.

Connections

Absolute value inequalities involving less than are often used to describe tolerance or accurateness of measurement. For example, in January 2002, USA Gymnastics declared that a vault table in its Junior Olympic program must have a maximum height of 125 cm, with a maximum allowable margin of error of 1 cm. This condition can be described by the absolute value inequality $|a - 125| \le 1$, where a is the actual height of the vault table.

Additional Examples

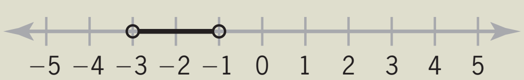
Solve and graph.

3. $3|x + 2| - 4 < -1$

Isolate the absolute value expression by first adding four to both sides and then dividing by three.

$$3|x + 2| - 4 < -1$$
$$\underline{\quad + 4 \quad + 4}$$
$$3|x + 2| < 3$$
$$\frac{3|x + 2|}{3} < \frac{3}{3}$$
$$|x + 2| < 1$$

$$-1 < x + 2 < 1$$
$$\underline{-2 \quad -2 \quad -2}$$
$$-3 < x < -1$$



4. $10 + 2|x + 1| \le 22$

Subtract 10 from both sides and then divide both sides by two.

$$10 + 2|x + 1| \le 22$$
$$\underline{-10 \qquad\qquad -10}$$
$$2|x + 1| \le 12$$
$$\frac{2|x + 1|}{2} \le \frac{12}{2}$$
$$|x + 1| \le 6$$

$$-6 \le x + 1 \le 6$$
$$\underline{-1 \quad -1 \quad -1}$$
$$-7 \le x \le 5$$