6.2 teacher notes

Objectives

- Solve absolute value equations in which the absolute value expression is isolated in one step.
- Solve absolute value equations in which the absolute value expression is isolated in two steps.

Vocabulary

Absolute value (Lesson 6-1)
Compound statement (Lesson 5-5)
Disjunction (Lesson 5-6)

Prerequisites

Solving equations of the form $|x| = k$ and $|ax + b| = k$

Solving one and two-step linear equations

Get Started

- Prepare 3 index cards so that each one is blank on one side and has the expression $|x - 5|$ written on the other side.
- On the board, show the class the equation 2 ▭ + 5 = 13 (use the blank side of one index card for the rectangle).
- Say, "The index card is hiding the unknown quantity. What steps should I take to isolate the index card?" Subtract 5 from both sides, then divide both sides by 2.
- Show the steps needed to isolate the index card on the left side. Use a new index card in each line.

  2 ▭ + 5 = 13
  2 ▭ = 8
  ▭ = 4

- Turn over all the index cards to reveal the expression $|x - 5|$.
- Point to the top equation ($2|x - 5| + 5 = 13$) and say, "To solve equations like this one, we will first need to isolate the absolute value expression using inverse operations, then write a disjunction and solve."

Section 1

Expand Their Horizons

In Section 1, students will solve absolute value equations in which one side of the equation contains not only an absolute value expression but also other operations. To solve these types of equations, the absolute value expression must first be isolated on one side.

Solving advanced absolute value equations requires steps similar to solving multi-step equations. To solve the equation $|x| - 1 = 5$, isolate the absolute value expression on the left side by adding one to both sides. Only when the equation is in the form $|\textit{linear expression}| = k$, the disjunction can be written and solved to find the solution.

1 Students may fail to isolate the absolute value expression before writing the disjunction. Given the equation $2 = |n| + 3$, they may write $2 = n + 3$ or $-2 = n + 3$. Emphasize that the equation must take the form $|\textit{linear expression}| = k$ before students can attempt to find solutions by writing a disjunction.

2 To solve, eliminate the coefficient of the absolute value expression by dividing both sides of the equation by –3. The result is $|a| = -4$. Because the equation shows an absolute value expression equal to a negative number, the equation has no solution.

Common Error Alert

Students may fail to identify that the absolute value expression is not isolated in the equation $-3|a| = 12$. It may help some students to re-copy the problem, replacing the entire absolute value expression with a symbol, such as a ★. This may help them see that the coefficient –3 must be eliminated from the left side of the equation $-3★ = 12$.

Additional Examples

1. $\mathbf{5 = 3 + |x|}$

Isolate the absolute value expression by subtracting three from both sides.

$$5 = 3 + |x|$$
$$-3 \quad -3$$
$$2 = |x|$$
$$x = 2 \text{ or } x = -2$$

2. $\mathbf{\frac{1}{2}|x| = -3}$

Isolate the absolute value expression by multiplying both sides by two.

$$2\left(\frac{1}{2}|x|\right) = -3(2)$$
$$|x| = -6$$
$$\varnothing$$

Section 2

Expand Their Horizons

In Section 2, students will be solving two-step linear equations that have absolute value expressions. Before solving the equations in this section, review how to solve two-step linear equations such as $3x + 2 = 11$. Remind the class of the following steps: isolate the x term ($3x$) by using an inverse operation to "undo" the addition of two; next, eliminate the coefficient on x by using an inverse operation to "undo" the multiplication by three. Remind them that when isolating a variable, "undo" operations by using the reverse order of operations.

Solve two-step absolute value expressions the same way. Instead of isolating the x term first, isolate the *absolute value expression* (which includes its coefficient). Then, eliminate the coefficient. Point out that the absolute value bars serve as a grouping symbol (like parentheses and brackets), and are, therefore, the last operation to be "undone" (by writing the disjunction equations).

3 This equation eventually takes the form $|\textit{linear expression}| = 0$. This means that the equation $n - 5 = 0$ is the only equation to be solved.

4 Students may distribute in the expression $2|x + 7| + 2$ to get $|2x + 14| + 2$. Remind them that the absolute value bars act as a grouping symbol and "protect" the operations inside them from operations outside them.

Connections

The notation $|x|$ was introduced by Karl Weierstrass in 1841. In many computer languages, the notation for $|x|$ is ABS(x). Challenge students to experiment with programming a computer or graphing calculator to find the absolute value of a number using the ABS notation.

Additional Examples

3. $\mathbf{|x - 4| - 5 = -2}$

Isolate the absolute value expression by adding 5 to both sides.

$$\begin{aligned} |x - 4| - 5 &= -2 \\ +5 \quad &\quad +5 \\ |x - 4| &= 3 \end{aligned}$$

$$\begin{aligned} x - 4 = 3 \text{ or } x - 4 &= -3 \\ +4 \;\; +4 \qquad +4 &\;\; +4 \\ x = 7 \text{ or } \quad x &= 1 \end{aligned}$$

4. $\mathbf{3 - 5|x + 1| = 13}$

First subtract 3 and then divide by –5

$$\begin{aligned} 3 - 5|x + 1| &= 13 \\ -3 \qquad &\;\; -3 \\ -5|x + 1| &= 10 \\ \frac{-5|x + 1|}{-5} &= \frac{10}{-5} \\ |x + 1| &= -2 \\ &\varnothing \end{aligned}$$

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