6.1 teacher notes

Objectives

- Understand the geometric and algebraic definitions of absolute value.
- Solve absolute value equations in the form $|ax + b| = k$ by rewriting it as a compound equation.
- Identify and solve one solution and no solution absolute value equations

Vocabulary

Absolute value (Lesson 1-2)
Coefficient (Lesson 2-1)
Solution set (Lesson 5-1)
Isolate (Lesson 3-1)

Prerequisites

Graphing points on a number line

Solving one-step linear equations

Solving two-step linear equations

Writing solution sets using set notation

Get Started

- Ask students to draw a number line from –10 to +10 on their own paper.
- Say, “On your number line, graph a point at any integer from –10 to +10.”
- Ask, “Who graphed a number that is 4 units away from 0?” Students who graphed +4 and those who graphed –4 should raise their hands. Ask the students who raised their hands to name the integers they graphed. (If both –4 and +4 are not responses, repeat the question with other distances from zero until you have two students who chose opposite integers.)
- Tell the class, “For any positive number *a,* there are always *two* numbers that are *a* units away from 0 — one to the left of 0, and another to the right of 0.”
- Ask, “How many numbers are zero units away from zero?” One. “What is that value?” 0.
- Can you have a negative distance from 0? No.

Section 1

Expand Their Horizons

In Section 1, students solve equations that include an absolute value expression. Review the definition of absolute value with the class. The absolute value of a number is its distance from 0 on a number line. Some students may be confused by the term "distance" in this definition. They may be thinking in terms of inches or centimeters. Remind the class that on a number line, distance is measured in the generic, *units,* and is always nonnegative. The distance from 0 to 1 is *one unit,* the distance from –3 to 8 is *11 units,* and so on.

Students may sometimes incorrectly simplify absolute value expressions by thinking "drop the negative from" or "change the sign of" the expression inside the absolute value bars. For example, they may say $|6| = -6$ or may simplify $|-4 + 3|$ as $|4 + 3|$. Remind students that absolute value and opposite have different meanings. Also, point out that absolute value symbols act like grouping symbols: you must simplify any expression within the absolute value symbols before finding the absolute value.

The measurement of distance is *always* nonnegative. Therefore, the absolute value of any number is almost always a positive number. The one exception is $|0|$, which is 0. Help students see that it is natural for distance to be nonnegative; in real life they would not express a distance as being negative. For example, "I live –3 miles from school" is not natural.

Encourage students to understand that $|5|$ is not only the "the absolute value of five", but also analogous to "the distance from 0 to 5." For visual learners, it may be helpful to read an equation like $|x| = 5$ as "the distance from 0 to x is five units" and to draw a number line, asking, "What numbers satisfy the condition of being five units from 0?"

The formal definition of absolute value may be confusing:

$$|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

Spend some time reviewing the definition with the class. The bracket notation indicates the two possibilities for values of x. Cover the top line and read the definition as "the absolute value of x is equal to x, if x is greater than or equal to 0". In other words, for 0 and positive numbers, the absolute value of a number is that number (e.g. $|10| = 10$). Next, cover the bottom line and read the definition as "the absolute value of x is equal to *the opposite of x,* if x is less than 0." In other words, for negative numbers, the absolute value of a number is the opposite of that number (e.g. $|-4| = -(-4) = 4$).

When writing the solution to an absolute value equation, students should be careful in their uses of the words "and" and "or." When listing the solutions, it is acceptable to say "The solutions are four and two" because *and* is used here as an English conjunction, not in the mathematical sense. However, when stating the solution, they must use the form "$x = 2$ or $x = 7$." The use of the word "or" (used here in the mathematical sense) implies that either value makes the equation true. Point out that the conjunction "$x = 2$ and $x = 7$" has no solution because x can not be equal to both two and seven and should not be used.

Review set notation with students. *Roster notation,* used in this lesson, uses braces to enclose a list of *elements* separated by commas. With roster notation, the set of factors of 12 is written as {1, 2, 3, 4, 6, 12}. Set builder notation uses the notation $\{x| \ x = 1, 2, 3, 4, 6, \text{ or } 12\}$ and is read "any *x,* such that *x* is equal to one, two, three, four, six, or twelve". When a set contains no elements, it is called *the empty set,* and can be written using the symbol ∅ or, {}. The notation {∅} is redundant and should not be used.

1 To solve the equation $|N| = 7$, have students read the equation as "the distance from N to zero is 7 units." There are two numbers that are 7 units from 0 on the number line (–7 and 7), so the solution set is $N = 7$ or $N = -7$.

2 By the time they solve this equation, students will probably recognize that there are two numbers that have an absolute value of 12 (12 and –12). Tell them that it is the *expression inside the absolute value bars* that must have a value of 12 or –12. When a value for y is chosen such that $4y = 12$ or $4y = -12$, the equation $|4y| = 12$ will be true. To find those values for y, solve each of the equations. Both equations can be solved by dividing both sides by four. The solutions are 3 and –3. The solution is $y = 3$ or $y = -3$.

3 Students may attempt to isolate the variable *z* before writing the disjunction. They may end up solving the equation $|z| = 2$. Although this method produces the correct answer in this case, tell students that an equation in the form |*expression*| = *k* (where *k* is a real number) needs to be solved by writing two equations before solving.

At this point, students may be ready to write a general rule for solving absolute value equations of the form |*expression*| = *k*. The solutions to such an equation can be found by solving the equations of the disjunction *expression* = *k* or *expression* = –*k*.

4 When solving the equation $|2l - 7| = 1$, point out that the equation is in the form |*expression*| = *k*, and can be separated just like previous equations. The only difference is that the equations of the disjunction will require two steps to isolate the variable, rather than one. Write the disjunction $2l - 7 = 1$ or $2l - 7 = -1$. Solve each equation by adding 7 to both sides, then dividing both sides by 2.

Common Error Alert

Students may be tempted to find a short-cut method for solving the equations of a disjunction. For example, they may solve one equation, find its solution, and claim that the other solution is the opposite of that solution.

Additional Examples

Solve.

1. $|x - 5| = 1$

Write two equations to solve this absolute value equation.

$$\begin{array}{rrl} x - 5 = 1 & \text{or} & x - 5 = -1 \\ +5 \quad +5 & & +5 \quad +5 \\ x = 6 & \text{or} & x = 4 \end{array}$$

2. $|2x + 8| = 2$

Write two equations to solve this absolute value equation.

$$\begin{array}{rcr} 2x + 8 = 2 & \text{or} & 2x + 8 = -2 \\ -8 \quad -8 & & -8 \quad -8 \\ 2x = -6 & & 2x = -10 \\ \frac{2x}{2} = \frac{-6}{2} & & \frac{2x}{2} = \frac{-10}{2} \\ x = -3 & \text{or} & x = -5 \end{array}$$

Section 2

Expand Their Horizons

In Section 2, students will encounter equations where the absolute value of an expression is equal to zero and equations where the absolute value of an expression is equal to a negative number. Remind students that 0 is neither positive nor negative, so $|x| = 0$ implies $x = 0$. There will be, at most, one solution in the solution set. For an absolute value of an expression which equals a negative number, remind students that the absolute value of an expression is always nonnegative. There is no solution for equations of this form.

5 Any equation that takes the form |*expression*| = negative number has no solution. Remind students that the absolute value of a number must always be nonnegative.

6 For any equation that takes the form |*linear expression*| = 0, the expression must be equal to 0. Such an equation has only one solution.

Additional Examples

Solve.

1. $|x - 2| = -3$

The absolute value of an expression is always nonnegative. The solution set is the null set.

The solution set is given by $\varnothing$.

2. $|5x + 10| = 0$

In this case, only one equation is written because only one number can satisfy the condition.

$$\begin{aligned} 5x + 10 &= 0 \\ -10 \quad &\;\; -10 \\ 5x &= -10 \\ \frac{5x}{5} &= \frac{-10}{5} \\ x &= -2 \end{aligned}$$