5.7 teacher notes

Objective

- **Write and solve inequalities of one variable to solve problems.**

Vocabulary

Perimeter (Lesson 4-4)
Base pay
Commission
Integer (Lesson 1-1)
Consecutive

Prerequisites

Solving one, two and multi-step linear inequalities

Translating word problems into number sentences

Solving word problems using a four-step process

Using perimeter

Using percent and decimal equivalents

Get Started

- Say, "Write an inequality for the following sentence: The sum of twice a number and 7 is at most 13." Write the sentence in words on the board and have each student write the mathematical expression for the inequality. $2x + 7 \leq 13$
- Ask, "What key words or phrases in the word problem indicate certain operations or symbols?" sum (addition); twice (multiplication); at most ($\leq$).
- Ask, "What other expressions might replace "at most" without changing the inequality? What other English expressions indicate that we should use the less than or equal to symbol?" no more than; less than or equal to; a maximum of; etc. Write student responses on the board.

Section 1

Expand Their Horizons

In Section 1, students will have to combine several skills. The four-step process requires that students read the problem, determine what to find, define variables, and write and solve an inequality. Students are already familiar with the four-step process from Module 4, in which students wrote and solved linear equations from word problems. Students will have to identify key words and phrases that indicate not only which operations to use, but also which of the four inequality symbols they will use. Students may want to create a table that indicates some common phrases and which inequality symbol should be used to translate the problem into an algebraic inequality.

For the first example, review the definition of *perimeter* with the class. The perimeter of a polygon is the sum of the lengths of its sides. Draw a square on the blackboard and label each side s, since each side of a square has the same length. Remind them that the perimeter of a square is $s + s + s + s$, and show them that the expression can be simplified to $4s$. It is the expression $4s$ that must be less than 120, leading to the inequality $4s < 120$. The symbol "$<$" is used in this inequality because of the key phrase "less than."

Take a moment to interpret the answer, $s < 30$. Ask students if a square sign measuring 30 inches on each side would be acceptable. Because the perimeter of such a sign would be $4(30) = 120$ inches, it would not be acceptable. Each sign's perimeter must be *less than* 120 inches.

1 In this question, the phrase "at least" appears. Help students write the correct inequality by writing the "total score is at least 46" on the board. Underline as shown, then replace each section with its mathematical equivalent. The total score can be represented by the expression $36.6 + s$; the phrase "is at least" can be replaced by the symbol "$\geq$"; and 46 remains 46.

Common Error Alert

Students may use the wrong inequality symbol. Encourage students to choose a test point in their solution set to check their answer. For example, a student who uses the symbol "$\leq$" in this example will get the answer $s \leq 9.4$. The test point 5 produces a total score of $36.6 + 5$, or 41.5, which does not satisfy the conditions of the example. This result tells the student to check that he or she used the correct inequality symbol.

In the second example, some students may question the formation of the inequality $25 - 2c \geq 9$. They may challenge that inequality with the inequality $9 + 2c \leq 25$, which shows that the 9 cups needed for the cakes, plus the amount used for the cookies ($2c$), must be less than or equal to the 25 cups available. Help students understand that this equation is valid, and point out that the two inequalities are equivalent. Add $2c$ to both sides of the first inequality to get the second.

2 This exercise uses the vocabulary words *base pay* and *commission*. Review these terms with students. *Base pay* is paid regardless of sales. *Commission* is determined as a percent of the salesperson's total sales. To find a salesperson's total pay, add the base pay and the commission. Use the expression $0.10s$ to represent the commission. Remind students that 0.10 is the decimal equivalent of 10%, and $0.10s$ means 10% of s.

3 In this exercise, consecutive integers are introduced. *Consecutive* means "in order", or "successive". Help students use the word *consecutive* by pointing out that Sunday, Monday and Tuesday are three *consecutive* days of the week. Likewise, 4, 5, and 6 are three *consecutive* counting numbers. Tell students that for consecutive integer

problems, they should call the least of the numbers x. If four is x, then five can be represented by the expression $x + 1$, six by $x + 2$, and so on. Point out that for each successive integer, the expression is increased by one (i.e. x, $x + 1$, $x + 1 + 1$, etc).

Common Error Alert

Students may represent the integers in consecutive integer problems incorrectly. For example, some students will model consecutive even integers correctly with x, $x + 2$, $x + 4$, etc., but will model consecutive odd integers incorrectly with x, $x + 1$, $x + 3$, etc. It may help to post a chart somewhere in the classroom showing the proper expressions to use for each type of problem.

Consecutive integers:

x, $x + 1$, $x + 2$, . . .

Consecutive even integers:

x, $x + 2$, $x + 4$, . . .

Consecutive odd integers:

x, $x + 2$, $x + 4$, . . .

To help students understand how to model a problem describing consecutive odd integers, write the integers five, seven, and nine on the board. Show students that if five is represented by x, then $x + 2$ represents seven, and $x + 4$ represents nine. Emphasize that consecutive even (or odd) integers are 2 units apart, so you must increase the expression by two each time.

Connections

After completing the problems in this lesson, students should be able to see how linear inequalities can be used in everyday situations. Many situations in which consumers must decide between two price options involve writing and solving inequalities. For example, one car rental agency may have a higher daily fee but a lower rate per mile, while another has a lower daily fee with a higher rate per mile. A linear inequality can help determine which option is more cost-effective.

Additional Examples

1. The sum of two consecutive even integers is greater than 85. What are the least possible values for the integers?

$$\begin{aligned} x + x + 2 &> 85 \\ 2x + 2 &> 85 \\ -2 \quad & \quad -2 \\ 2x &> 83 \\ \frac{2x}{2} &> \frac{83}{2} \\ x &> 41.5 \end{aligned}$$

The least possible values for the integers are 42 and 43.

2. Talk-a-lot long distance service has a monthly fee of \$4.95 and charges \$0.08 per minute talked. PhoneMore long distance service has a monthly fee of \$7.95 and charges \$0.05 per minute talked. For how many minutes talked is PhoneMore more expensive than Talk-a-lot?

$$\begin{aligned} 4.95 + 0.08x &< 7.95 + 0.05x \\ -4.95 \quad & \quad -4.95 \\ 0.08x &< 3 + 0.05x \\ -0.05x \quad & \quad -0.05x \\ 0.03x &< 3 \\ \frac{0.03x}{0.03} &< \frac{3}{0.03} \\ x &< 100 \end{aligned}$$

PhoneMore is more expensive if the number of minutes talked is less than 100 minutes.