5.6 teacher notes

Objective

- Solve and graph the solution sets to disjunctions.

Vocabulary

Compound inequality (Lesson 5-5)
Conjunction (Lesson 5-5)
Disjunction
Union (Lesson 1-1)
Intersection (Lesson 1-1)
Real number (Lesson 1-1)

Prerequisites

Solving conjunction inequalities

Solving one-step and two-step linear inequalities

Graphing inequalities on a number line

Get Started

- Say to the class, "Write a number down on your paper. It can be any number you like, positive, negative, fraction, or decimal."
- Say, "Listen carefully to the following condition. If your number is either *greater than –10* ***or*** *less than 10,* I will write it on the board."
- While students think, draw a number line on the board from –15 to 15. Ask a student to tell you his or her number. Graph the number in colored chalk.
- Students should realize that all their numbers are either greater than –10 or less than 10.
- Say, "Any real number meets the condition of being either greater than –10 or less than 10."

Section 1

Expand Their Horizons

In this lesson, students study a second type of compound inequality, the disjunction. A disjunction inequality consists of two inequalities (in this case linear inequalities) joined by the word or.

Review the mathematical meaning of the word *and* with students. A conjunction inequality consists of two inequalities joined by the word *and*. In order to be in the solution set of a conjunction inequality, a number must be a solution to both the first inequality *and* the second inequality.

In order to be in the solution set of a disjunction inequality, a number needs to only be in the solution set of either or both inequalities. That is, a number can be in the solution set of the first inequality *or* in the solution set of the second inequality. The number could also be in the solution sets of both inequalities.

In the lesson example, the condition necessary to be in the solution set of a disjunction inequality as needing to be in "either or both" solution sets. Offer the alternative phrase "at least one." To be in the solution set of the disjunction inequality, a number must be in the solution set of *at least one* of the inequalities. A number is in *at least one* of the solution sets if it is in one *or* in both solution sets.

Help students understand the difference between the mathematical use of the words *and* and *or*. Recall the scenario from the previous lesson, in which Newt was not admitted into Club Roxy because he did not meet the criteria for admission: breathing *and* being a super-great dancer. The conditions formed a conjunction. If the word *or* were to replace the word *and,* Newt would be admitted (recall that Newt is breathing but is not a super-great dancer). Since Newt meets at least one of the requirements for admission, he would be allowed in under the conditions of a disjunction. Have students think of other real-life situations for which a consequence is achieved after satisfying at least one of the conditions of a disjunction.

Visual learners may find the use of color helpful here. Have students draw the graphs of both solution sets with different colors. The solution to the disjunction inequality is any number graphed.

Common Error Alert

Students may attempt to write the solution set using a notation such as $0 \geq x \geq 4$. Remind students that this compound inequality notation is only to be used for conjunctions. Also point out that this notation states that $0 > 4$, which is false.

Review the definition of *union* with students. For two sets A and B, the union of the sets is a set made up of the elements of A and the elements of B, with repeated elements listed only once. So, the union of sets $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$ is written $A \cup B = \{1, 2, 3, 4, 6, 8\}$. Students are probably familiar with the word union from everyday uses. The union of two entities is the result of their joining or coming together. A disjunction inequality is a union of two inequality statements.

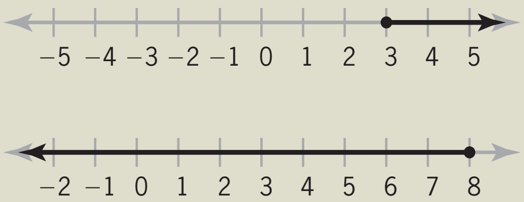
1 Any number less than three is less than five, so the set $x < 3$ is also included in the set $x < 5$. The solution set is given by $x < 5$.

2 Remind students to be careful about the type of circle they draw when graphing each solution set. Both graphs for this disjunction use open circles. Five is the only number not included in at least one of the solution sets. To express that the solution set of the disjunction is the set of all real numbers except five, write the inequality $x \neq 5$.

3 The solution sets to the inequalities $x \geq 4$ or $x \leq 0$ have no elements in common. Their union, therefore, is the set whose elements are all the elements of the first set, and also all the elements of the second set. In this case, the solution set looks exactly like the statement of the disjunction itself.

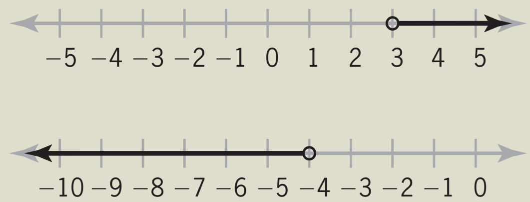
Additional Examples

1. $x \geq 3$ **or** $x \leq 8$



The solution set is given by: $\Re$

2. $x > 3$ **or** $x < -4$



The solution set is given by: $x > 3$ or $x < -4$

Section 2

Expand Their Horizons

In Section 2, students will solve disjunction inequalities and solve. Review the steps for solving linear inequalities. First, simplify on each side if necessary. Next, isolate the variable term using inverse operations of addition or subtraction. Finally, isolate the variable by using inverse operations of multiplication or division to eliminate the coefficient of the variable. When multiplying or dividing both sides of an inequality by a negative number, the direction of the inequality sign must be reversed.

4 Solving this disjunction requires that each of its inequalities first be solved for x. To solve $x + 6 \leq 2$, subtract six from both sides to get $x \leq -4$. To solve $4x \leq -8$, divide both sides by four to get $x \leq -2$. Once both answers are found, graph each solution set on a number line. Because all numbers less than −4 are less than −2, the union of the sets is $x \leq -2$.

Look Beyond

Conjunction and disjunctions and the related concepts of unions and intersections are useful concepts in many mathematics branches including set theory, logic, linear programming, abstract algebra, and others.

Additional Examples

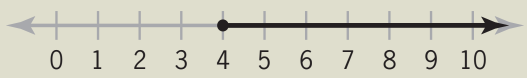
1. Solve and graph.

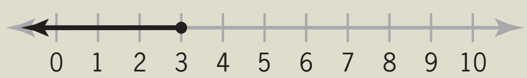
$x - 1 \geq 3$ or $-5x \geq -15$

$x - 1 \geq 3$ or $\frac{-5x}{-5} \geq \frac{-15}{-5}$

$\underline{+1} \quad \underline{+1}$

$x \geq 4$ or $x \leq 3$





The solution set is given by: $x \geq 4$ or $x \leq 3$

2. Solve and graph.

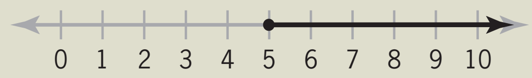
$2x - 4 \geq 6$ or $4 - x \leq 3$

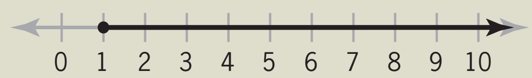
$2x - 4 \geq 6$ or $4 - x \leq 3$

$\underline{+4} \quad \underline{+4} \qquad \underline{-4} \quad \underline{-4}$

$\frac{2x}{2} \geq \frac{10}{2}$ or $\frac{-x}{-1} \leq \frac{-1}{-1}$

$x \geq 5$ or $x \geq 1$





The solution set is given by: $x \geq 1$