5.5 teacher notes

Objectives

- Solve and graph the solution sets to conjunctions.
- Use the notation $a < x < b$ to show that x lies between a and b, $a < b$.

Vocabulary

Linear inequality (Lesson 5-1)
Compound inequality
Conjunction
Intersection (Lesson 1-1)
Empty set (Lesson 1-1)
Solution set (Lesson 5-1)
Isolate (Lesson 3-1)

Prerequisites

Solving one, two, and multi-step linear inequalities

Graphing inequalities

Get Started

- Have students get into pairs to play a "Betweenness" Game. Write the statement,"There is at least one real number that is greater than ________ and less than ________", on the board. Have them write the numbers from –10 to 10 on individual index cards and shuffle them. One student will choose to be "true" and the other will be "false".
- For each turn, students will deal two cards. The first number goes in the first blank and the second number goes in the second blank. For example, if a 5 and a –2 are dealt the statement would be: There is at least one real number that is greater than 5 and less than –2.
- If the statement is true, the student who chose "true" gets a point. Similarly, if the statement is false, as in the statement above, the student who chose "false" will get to a point.
- Continue until one player earns 10 points; the student who is first to earn 10 points is the winner. Cards can be shuffled and reused to finish the game.

Section 1

Expand Their Horizons

In Section 1, students are introduced to the mathematical meaning of the word *and*. After defining a conjunction inequality, take a few moments to discuss the meaning of the word *and*.

Students are obviously familiar with non-mathematical uses of the word *and*. In English, the word *and* simply joins other words together (e.g. running and jumping; teachers and students; red, white, and blue). Point out that as a part of speech, *and* is classified as a *conjunction*. In English, conjunctions connect words and phrases. Emphasize that when used to join two mathematical statements as part of a *conjunction inequality,* and has special meaning.

Ask students to think of scenarios where two conditions must be met in order for a consequence to occur. For example, in order to attend a certain field trip, a student must pay $5.00 and turn in a permission slip. Point out that neither condition alone is enough to qualify the student to go; both conditions must be met.

Likewise, in the conjunction inequality $x < 0$ and $x \leq 4$, a number must satisfy both conditions to be part of the solution set. Write the example on the board, solve, and draw a number line. Draw the solution sets to each inequality. Then, starting at the left end of the number line and moving to the right, stop at each integer and ask, "Is this number less than zero? Is it less than or equal to four?" If either answer is no, the integer does not lie in the solution set of the conjunction. Pay close attention to the integer zero. Ask, "Is zero less than zero?" Be sure students recognize why zero is not in the solution set.

For visual learners, it may be helpful to use color to reinforce how to determine solution sets from graphs. Have students use one color to do the shading for the first inequality, and another color to do the shading for the second inequality. Show students that the where the colors overlap is the solution set. Point out that they will have to pay special attention to the endpoints of each interval when determining the final solution set.

1 After the conjunction inequality $x \geq 6$ and $x \leq 2$ is solved, have students explain why this conjunction inequality has no solution. Then, ask them to write another conjunction inequality that has no solution. Have them compare with a classmate and confirm that there is no solution.

Review the meaning of the symbol $\varnothing$ with students. The symbol means "empty set". That is, the solution set has no elements. Remind students that the {} can also be used to show an empty set.

Review with students the definition of *intersection*. For two sets A and B, the intersection of the sets is the set made up of the elements that appear in both sets A and B. So, the intersection of sets $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$ is written $A \cap B = \{2, 4\}$. A conjunction inequality is an intersection of two inequality statements.

Common Error Alert

Compound inequalities of the form $a < x < b$ are sometimes misused by students. For example, students may write $-1 > x > 3$ to represent the answer to the second example. Point out that such a statement implies that $-1 > 4$, which is a false statement. Remind students that in a "compound inequality", both inequality signs must point in the same direction, and that the three values used in the statement must be written in order of value according to that comparison.

2 Remind students that they can use the shorthand notation $1 < x < 4$ to show that the solution includes all the numbers **between** one and four.

3 In this case, the graphs of the two inequalities of the conjunction point in the same direction. Some students may notice that when this happens, the solution set is always identical to one of the inequalities, in this case $x \leq -2$.

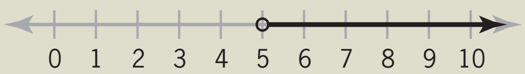
Additional Examples

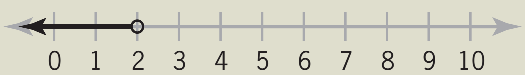
1. Solve and graph.

$x > 5$ and $x < 2$

There is no area of intersection for the two inequalities.

$\varnothing$



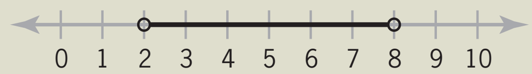


2. Solve and graph.

$x \geq 2$ and $x < 8$

The section of the number line including 2 and up to 8 shows the intersection of the two inequalities.

$2 \leq x < 8$



Section 2

Expand Their Horizons

For Section 2, students are simply combining skills from previous lessons with the idea of conjunction inequalities. They must solve each inequality before attempting to determine the solution set to the conjunction inequality.

Review the procedure for solving inequalities. First, simplify each side if necessary. Next, move all the variable terms to one side and all the constant terms to the other side. Finally, eliminate the coefficient of the variable using multiplication or division. Remind students to reverse the direction of the inequality when multiplying or dividing by a negative number.

To solve conjunction inequalities of the form $-2 < x + 6 < 10$, students will have to change their thinking and their language to reflect *three parts* instead of *two sides*. To solve this inequality, subtract six from *all three parts*. Encourage students to show the subtraction of six three times.

4 This conjunction inequality is made up of a two-step inequality and a one-step inequality. Point out this difference and review the steps for solving each type. Remind students that they must reverse the direction of the second inequality when dividing by –3.

5 For students having trouble solving compound inequalities, have them break the conjunction inequality into two individual inequalities ($0 \leq x + 2$ and $x + 2 \leq 8$). Have them solve each inequality and graph the solution sets. Then point out that the intersection of the graphs is the region between –2 and 6.

Connections

Now that students have been introduced to the special mathematical meaning of *and,* they will no doubt find themselves seeing situations that can be modeled with a conjunction inequality in their everyday lives.

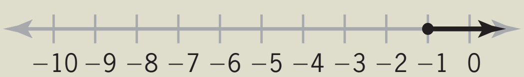
For example, they may notice the admission prices at a movie theatre are broken down by age. People under 12 *and* over 65 pay a discounted rate; everyone 12 and over, *and* 65 and younger pays the normal rate. If the person's age is *a,* then the normal rate is paid when $12 \le a \le 65$.

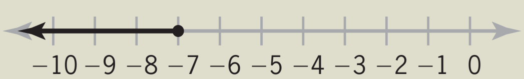
Additional Examples

1. Solve and graph.

$$3x + 6 \ge 3 \quad \text{and} \quad -x - 5 \ge 2$$
$$\underline{-6 \;\; -6} \qquad \underline{+5 \;\; +5}$$
$$\frac{3x}{3} \ge \frac{-3}{3} \quad \text{and} \quad \frac{-x}{-1} \ge \frac{7}{-1}$$
$$x \ge -1 \quad \text{and} \quad x \le -7$$

$\varnothing$





2. Solve and graph.

$$-4 \le 6 - 2x < 2$$
$$\underline{-6 \;\; -6} \qquad \underline{-6}$$
$$\frac{-10}{-2} \le \frac{-2x}{-2} < \frac{-4}{-2}$$
$$5 \ge x > 2$$