5.4 teacher notes

Objective

- **Solve and graph the solution sets to inequalities with variables on both sides.**

Prerequisites

Solving inequalities with variables on one side

Graphing inequalities

Simplifying expressions with integers; using the Distributive Property

Combining like terms

Vocabulary

Linear inequality (Lesson 5-1)
Variable (Lesson 2-1)
Constant (Lesson 2-1)
Isolate (Lesson 3-1)
Inverse operations (Lesson 3-3)
Simplify (Lesson 1-2)

Get Started

- Write the equation $4(x - 2) = 6x + 12$ on the board.
- Ask, "What is the first step in solving this equation?" Simplify the left side.
- Ask, "What property is needed?" The Distributive Property of Multiplication Over Addition
- Below the original equation, write the equation $4x - 8 = 6x + 12$.
- Ask, "What is the next step?" Move all the variable terms to one side and all the other terms to the other side.
- Below the previous equation, write the equation $-20 = 2x$. Ask students to verify this work on their own papers.
- Ask, "What is the final step?" Divide both sides by 2 to solve.
- Below the previous equation, write the equation $-10 = x$.
- On the board, write the heading "Steps for Solving". Ask students to identify the steps they used to solve the equation. Simplify each side, move variable terms to one side, move constant terms to the other side, eliminate the coefficient of the variable.

Section 1

Expand Their Horizons

In Section 1, students will solve multi-step inequalities. Before you begin the lesson, tell them that they already have all the skills necessary to solve the equations in this lesson. Explain to them that they will put together several different skills they have already learned.

Review with students that when solving an inequality, the goal is to isolate a single variable term on one side of the inequality and a constant term on the other. To do this, students must simplify each side of the inequality, use the Properties of Inequality to get variable terms on one side of the inequality and constant terms on the other side. Then they must use the reverse order of operations to isolate the variable. Point out that it is the student's choice to determine which side the variable terms are placed. When solving the inequality $5x - 4 > 3x$, show with the class how the inequality can be solved by moving the x terms to the left or to the right.

LEFT: $5x - 3x - 4 > 3x - 3x$

$2x - 4 + 4 > 4$

$2x > 4$

$x > 2,$

RIGHT: $5x - 5x - 4 > 3x - 5x$

$-4 > -2x$

$2 < x.$

Both methods are valid and produce the same answer.

When solving an inequality like $2x - 4 \geq 5x + 8$, the student again has the choice of moving the variable terms on the right or on the left. Suggest that they solve this equation by subtracting $2x$ from both sides.

As you go through the first two examples in the lesson, discuss the choices students made in deciding which side of the inequality the variable should be. Find two students who used different methods and ask them to show their solutions on the board. Point out that both methods produced the same answer, although one method produced a negative coefficient.

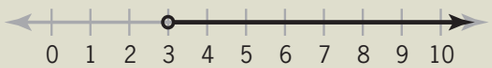
1 Note that to solve this problem, students could have also subtracted eight from both sides first. This would have made x's coefficient -14, and then students would have had to divide by a negative number and reverse the inequality symbol while solving. Adding $6x$ to both sides is easier.

2 For this item, students have to add both a variable term and a constant term to each side of the inequality to solve. Students may want to do both processes in one step, but this may cause confusion and careless errors. Have students write out every step.

Additional Examples

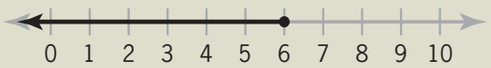
1. Solve and graph.

$$\begin{aligned} 3x - 10 &> -1 \\ +10 \quad &+10 \\ 3x &> 9 \\ \frac{3x}{3} &> \frac{9}{3} \\ x &> 3 \end{aligned}$$



2. Solve and graph.

$$\begin{aligned} 4x - 5 &\le 2x + 7 \\ -2x \quad &\quad -2x \\ 2x - 5 &\le 7 \\ +5 \quad &+5 \\ 2x &\le 12 \\ \frac{2x}{2} &\le \frac{12}{2} \\ x &\le 6 \end{aligned}$$



Section 2

Expand Their Horizons

Before starting Section 2, it may be necessary to review the Distributive Property of Multiplication Over Addition. Simplify a few expressions like $4(x - 3)$. Then, move to an expression like $4 - 3(x + 4)$. It may be helpful to replace the subtraction with addition, making the expression $4 + -3(x + 4)$. This change may make it easier for some students to avoid a sign mistake in their simplified expression. If necessary, write the intermediate step $4 + (-3)(x) + (-3)(4)$ before writing the expression $4 - 3x - 12$.

If students are having trouble using a series of steps to solve multi-step inequalities, designate a section of the blackboard or make a poster listing the steps. Enlist the students' help in outlining the steps necessary to solve. Your list may look something like this:

1. Simplify each side. Distribute, combine like terms.
   - Simplify the left side.
   - Simplify the right side.
   - Each side should now look like $a + bx$, where a and b are constants.
2. Use the properties of inequality to get all the constant terms on one side.
3. Use the properties of inequality to get all the variable terms on the other side. (Steps 2 and 3 are interchangeable)
4. Eliminate the coefficient of the variable using inverse operations (multiply or divide). Remember to reverse the direction of the inequality if the coefficient is negative.

For practice, have students identify each step when solving an inequality.

3 Students may need to rewrite the inequality as $8 + (-2)(3 = x) > 16 + (-1)(x + 2)$ so that they can better see what numbers need to be multiplied across the grouping symbols.

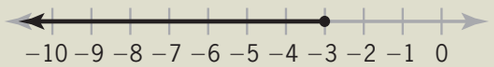
Connections

Inequalities are used in many applications. For example, a computer programmer may write a program in which the computer compares two expressions and executes a certain command only when one expression is greater than the other. Students who have experience in programming will be familiar "if-then" and "if-else" commands.

Additional Examples

1. Solve and graph.

$$3 - 2(x + 5) \geq 3(x + 1) + 5$$
$$3 - 2x - 10 \geq 3x + 3 + 5$$
$$-7 - 2x \geq 3x + 8$$
$$\underline{+ 2x} \quad \underline{+ 2x}$$
$$-7 \geq 5x + 8$$
$$\underline{- 8} \quad \underline{- 8}$$
$$-15 \geq 5x$$
$$\frac{-15}{5} \geq \frac{5x}{5}$$
$$-3 \geq x$$



2. Solve and graph.

$$3(4 - x) + 2 > 2x - (x + 6)$$
$$12 - 3x + 2 > 2x - x - 6$$
$$14 - 3x > x - 6$$
$$\underline{+ 3x} \quad \underline{+ 3x}$$
$$14 > 4x - 6$$
$$\underline{+ 6} \quad \underline{+ 6}$$
$$20 > 4x$$
$$\frac{20}{4} > \frac{4x}{4}$$
$$5 > x$$