5.1 teacher notes

Objectives

- Solve a linear inequality by inspection.
- Write the solution to an inequality in proper notation.
- Graph the solution for an inequality on a number line.
- Check the solution to an inequality.

Vocabulary

Linear inequality
Greater than
Less than
Greater than or equal to
Less than or equal to
Not equal to

Prerequisites

Graphing points on a number line

Evaluating expressions

Get Started

- Write the number sentence 2 + 3 _____ 5 on the board.
- Ask, "What symbol can be used in the blank to produce a true statement?" =
- Tell the class, "An **equation** is a number sentence that shows that two expressions are equal or have the same value. In this case, two plus three **equals** five."
- Write the number sentence 2 + 3 _____ 10 on the board.
- Ask, "What symbols can be used in the blank to produce a true statement?" $<$, $\leq$, or $\neq$
- Write the number sentence 2 + 3 _____ 1 on the board.
- Ask, "What symbol can be used in the blank to produce a true statement?" $>$, $\geq$, or $\neq$

Section 1

Expand Their Horizons

In Section 1, students are introduced to algebraic inequalities. An algebraic inequality is a statement formed by placing an inequality symbol between two expressions.

Discuss the root of the word **inequality** with the class. The prefix 'in' means 'not'. Students are familiar with other words with the prefix 'in'. For example, the words *in*dependence, *in*ability, and *in*complete contain the prefix 'in'. Point out to students that the symbols $\leq$ and $\geq$ include the possibility that two expressions could be equal. The mathematical symbols for inequalities include $<$, $>$, $\leq$, $\geq$, and $\neq$.

Review the definition of a *linear inequality of one variable* with the class. *Linear* means that the variable has a power of one ($x^1 > 0$ means $x > 0$). Tell students that in future courses they will study inequalities that are non-linear, e.g. quadratic inequalities, where the exponent of x is 2.

Common Error Alert

Students may be careless with the use of open and closed circles to indicate the endpoint of a graph. Remind students that equations are graphed on a number line with closed circles, so an inequality with $\geq$ or $\leq$ has a closed circle as an endpoint.

To solve by *inspection* means to solve using mental math. Contrast this method to solving equations using algebraic properties. Let students know they will be learning an algebraic method for solving linear inequalities of one variable in future lessons. Students may need extra practice seeing that the inequality $x > 3$ is the same as $3 < x$. Point out that as long as the symbol is "pointing" to the expression with the lesser value, the inequality is true.

An inequality has an infinite number of solutions. The solution set of $x > 3$ has as its elements all real numbers greater than three, including rational numbers (fractions, decimals, and integers) and irrational numbers. A graph of an inequality is represented with lines, circles, and arrows. For $x > 3$, there is one endpoint, in this case an open circle at three. Then, draw an arrow to the right of the open circle. The open circle shows that the graph contains all the numbers to the right of three, but not 3. The arrow on the right end of the line indicates that the solution set includes *all* numbers greater than three, no matter how large.

1. Offer students the following method to solve this inequality: To solve the inequality $x + 4 > 0$, mentally replace the *greater than* sign with an equal sign. The answer to that equation is the endpoint of the ray, $x = -4$. -4, is not a solution to the inequality, so draw an open circle at -4 on the number line. The ray extends in only one direction, so choose a test point on either side of -4 to determine whether the chosen point is in the solution set of the inequality. For example, choosing -10 gives $-10 + 4 > 0$, which is false. Choosing 10 gives $10 + 4 > 0$, a true statement. Shade to the right, showing that the solution set continues infinitely to the right.

2. Remind students that because part of the equal sign is in the inequality symbol, the solution to the equation $x + 2 = 0$ is part of the solution set of the inequality.

Look Beyond

In future lessons, students will learn algebraic methods for solving linear inequalities of one variable. The properties of equality allow students to operate on both sides of an equation to isolate the variable and solve. Properties of inequality allow students to do the same thing to inequalities.

Connections

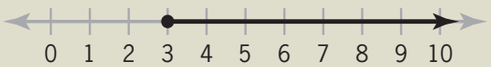
Linear inequalities can be used to model a variety of everyday situations. A student who must maintain a positive balance in a savings or checking account must solve an inequality. For example, the inequality $x - 12 > 0$ could be used when writing a \$12 check where x is the balance in the checking account.

Additional Examples

1. $x - 3 \geq 0$

- Find the endpoint $x - 3 = 0$ or $x = 3$.
- The inequality is $\geq$, so the circle will be closed.
- Shade to the right of the circle.

$x \geq 3$



2. $x + 1 < 0$

- Find the endpoint: $x + 1 = 0$ or $x = -1$.
- The inequality is $<$, so the circle will be open.
- Shade to the left of the circle.

$x < -1$