4.4 teacher notes

Objective

- Write and solve equations of one variable to solve mixture and rate problems.

Vocabulary

Distributive Property (Lesson 2-3)
Like terms (Lesson 2-4)

Prerequisites

Translating sentences into algebraic equations

Solving multi-step linear equations

Finding the percent of a number

Using the distance/rate/time formula

Get Started

Help students reach a better understanding of mixture problems by using this manipulative activity.

- You will need a bag of brown beans and a bag of white beans. Give each student 10 of each type of bean.
- Start with a mixture that is 40% white beans. Have students get two white beans and three brown beans. Remind them that two out of five is 40%.
- Ask, "How can you make the mixture have 70% white beans without removing any of the brown beans?"
- Allow students to investigate adding white beans until they arrive at an answer. Adding one white bean will make the mixture have $\frac{3}{6}$ or 50% white beans. When they have added five more white beans, the mixture will have seven white beans and three brown beans. It will be a 70% white beans mixture.

Section 1

Expand Their Horizons

In Section 1, students will solve basic mixture problems. These problems are very prevalent in chemistry courses. Use this opportunity to bring the study of science into the mathematics classroom.

After reading the problem and identifying what must be found, the next step in any type of application problem is to assign a variable to represent an unknown.

Check each answer. $d = 5$. $2d = 10$. $1.25(10) + 1.50(5) = 12.50 + 7.50 = 20.00$. The answers are correct.

1 Remind students to consider the value of each type of coin when working mixture problems with money. Once the variable is assigned, each denomination of coin should be represented using the variable. Then an equation can be written. Rick has all nickels and dimes. He has three less nickels than twice the number of dimes. Let d = number of dimes. Then, $2d - 3$ = number of nickels. The total value of these coins is \$3.85.

$0.05 \times$ the number of nickels $+ 0.10 \times$ the number of dimes $= 3.85$

$0.05 \times (2d - 3) + 0.10 \times d = 3.85$

Solve this equation to find that Rick has 37 nickels and 20 dimes.

Common Error Alert

When solving coin mixture problems, students will leave off the value of the coin. Some students may write the equation for Guided Notes example 1 as $2d - 3 + d = 3.85$. Point out that $2d - 3 + d$ represents a number of coins, not an amount of money. The value of a given number of nickels is found by multiplying by 0.05. The value of a given number of dimes is found by multiplying by 0.10.

For problems about solutions, like Ferd's gravy problem, the initial amount of an ingredient plus the added amount of that ingredient must equal the final amount of that ingredient. In Ferd's gravy problem, the amount of gravy in the 10% solution plus the amount of pure gravy that is added must equal the amount of gravy in the final solution.

Some people prefer to solve solution problems using a table. Ferd has 20 ounces of 10% gravy and wants to know how much pure, or 100%, gravy to add. He wants a solution that is 15% gravy. Let w represent the amount of pure gravy that must be added to reach the desired solution.

Use this table to write the equation.

	Amount of Solution	Amount of Gravy
10% solution	20	20(0.10)
100% solution	w	w
15% solution	$20 + w$	$0.15(20 + w)$

The amount of gravy in 10% solution + amount of 100% gravy = amount of gravy in 15% solution.

$$20(0.10) + w = 0.15(20 + w)$$

2 Use this chart to write the equation.

	Amount of Solution	Amount of Acid
50% solution	50	50(0.50)
25% solution	a	$0.25a$
40% solution	$50 + a$	$0.40(50 + a)$

Amount of acid in 50% sol. + amount of acid in 25% sol. = amount of acid in 40% sol.

$$50(0.50) + 0.25a = 0.40(50 + a)$$

Solve the equation. Dr Gonzales must add $33\frac{1}{3}$ mL of the 25% acid solution to obtain a 40% acid solution. Be sure that students include units in their final answers.

Additional Examples

1. A grocer mixes two kinds of nuts to form 25 pounds of a mixture worth \$3.00 a pound. One kind of nut is worth \$2.40 a pound and the other is worth \$3.30 a pound. How many pounds of each nut should the grocer mix to get the desired mixture of nuts worth \$3.00 a pound?

p = pounds of nuts at 2.40 a pound
$25 - p$ = pounds of nuts at 3.30 a pound

	Pounds of nuts	Cost of nuts
\$2.40 per pound	p	2.40p
\$3.30 per pound	$25 - p$	$3.30(25 - p)$
\$3.00 per pound	25	3(25)

$$2.40p + 3.30(25 - p) = 3(25)$$
$$2.40p + 82.5 - 3.30p = 75$$
$$-0.9p + 82.5 = 75$$
$$-0.9p = -7.5$$
$$p = 8\tfrac{1}{3}$$
$$25 - p = 16\tfrac{2}{3}$$

The grocer should use $8\frac{1}{3}$ pounds of the nuts that cost \$2.40 a pound and $16\frac{2}{3}$ pounds of the nuts that cost \$3.30 a pound.

2. How many milliliters of 30% ethanol must be mixed with 12 mL of 40% ethanol in order to obtain a solution that is 36% ethanol?

x = mL of 30% solution

	mL of Solution	mL of ethanol
40% solution	12	12(0.40)
30% solution	x	0.30x
36% solution	12 + x	0.36(12 + x)

$$12(0.40) + 0.30x = 0.36(12 + x)$$
$$4.8 + 0.30x = 4.32 + 0.36x$$
$$0.48 + 0.30x = 0.36x$$
$$0.48 = 0.06x$$
$$8 = x$$

8 milliliters of the solution that is 30% ethanol should be added to 12 milliliters of solution that is 40% ethanol to obtain a solution that is 36% ethanol.

Section 2

Expand Their Horizons

In Section 2, students will solve rate problems that use the formula, distance = rate · time. Because of the difficulty level of this lesson, it may be necessary to allow one day for each of the two sections in this lesson.

Have students sketch a picture for each problem before beginning to solve the problem. Have them use arrows to indicate the direction of travel by each vehicle.

The information for the first example can be summarized in a chart.

	Rate	Time	Distance
Train A	55	t	$55t$
Train B	75	t	$75t$

Because the trains are traveling away from each other, the distance between the trains is the sum of the distances that the two trains have traveled. The question asks how many hours it will be before the trains are 845 miles apart. This can be found by the equation $55t + 75t = 845$.

The second example also asks the students to find time. However, the time for each leg of the trip is not necessarily the same. The entire trip took $3\frac{1}{2}$ hours, so the number of hours spent driving 60 mph can be represented by t and the number of hours spent driving at 30 mph can be represented by $3\frac{1}{2} - t$. The total distance driven is 180 miles. Therefore, $60t + 30(3\frac{1}{2} - t) = 180$.

3 In order for Josh to catch Joel, the distance that each boy travels must be the same: the distance traveled by Joel = the distance traveled by Josh.

	Rate	Time	Distance
Joel	704	t	$704t$
Josh	880	$t - 3$	$880(t - 3)$

$704t = 880(t - 3)$ is the equation students should solve to answer the problem.

Connections

Chemists use their knowledge of solving mixture problems extensively. Various experiments require different percent solutions, but it is not efficient to manufacture solutions ranging from 1% to 100%. Instead, increments such as 20%, 50%, and 100% are manufactured, and chemists must mix or dilute these standard solutions to create specific percent solutions.

Look Beyond

In future lessons students will learn to solve mixture and distance/rate/time problems by using systems of equations. These systems of equations will consist of at least two variables to represent the unknown amounts.

Additional Examples

1. Marty and Luke went on a trip. The first day they averaged 48 miles per hour. On the second day they averaged 56 miles per hour and drove for 3 hours more than the first day. The combined distance for both days was 688 miles. How many hours did they drive altogether?

t = time day one
$t + 3$ = time day two

$$48t + 56(t + 3) = 688$$
$$48t + 56t + 168 = 688$$
$$104t + 168 = 688$$
$$104t = 520$$
$$t = 5$$

Total time: $t + (t + 3) = 5 + 5 + 3 = 13$

Marty and Luke drove 13 hours altogether.

2. Mr. and Mrs. Martinez work in towns that are 198 miles apart. If they both leave work and drive toward each other, they will meet in 2 hours 15 minutes. Mr. Martinez drives 4 m.p.h. faster than Mrs. Martinez. Find each person's average rate of speed.

r = Mr. Martinez's rate
$r - 4$ = Mrs. Martinez's rate

	Rate	Time	Distance
Mr. M	r	2.25	$2.25r$
Mrs. M	$r - 4$	2.25	$2.25(r - 4)$

$$2.25r + 2.25(r - 4) = 198$$
$$2.25r + 2.25r - 9 = 198$$
$$4.5r - 9 = 198$$
$$4.5r = 207$$
$$r = 46$$

Mr. Martinez's average rate of speed is 46 mph and Mrs. Martinez's average rate of speed is 42 mph.