4.3 teacher notes

Objective

- **Write and solve linear equations of one variable to solve geometry problems about perimeter and angle sums.**

Prerequisites

Translating sentences into algebraic equations

Solving multi-step linear equations

Finding the perimeter of a polygon

Vocabulary

Perimeter
Sum (Lesson 1-2)
Isosceles triangle
Congruent
Legs
Base
Like terms (Lesson 2-4)
Scalene triangle
Complementary angles
Supplementary angles
Vertex angle
Base angle

Get Started

- Use this activity to demonstrate the usefulness of a picture or model. Say, "Please follow these directions exactly. Take out a piece of paper, fold it in half vertically and then unfold it. Next, fold the top right corner to the center and fold the top left corner to the center. Then, fold the paper in half vertically once more. Finally, holding the paper with the fold facing downward, fold each top side outward to the fold."
- When you finish giving the directions, many students will likely be lost. Give the directions again, except this time demonstrate making a paper airplane as you give the directions.
- Explain that many times it is not enough to have words to follow. Very often a difficult task can be made much simpler by having a picture or model to follow.

Section 1

Expand Their Horizons

In Section 1, students will focus on the application of algebra in geometry problems. Some students may not be familiar with the geometry terms presented in this lesson. Take some time to allow students to become familiar with these terms. If necessary, they can create flash cards with the term on one side and a definition and sketch of the figure(s) on the other side.

One way to describe perimeter is to say that it is the distance around a polygon. If there are 20 feet of fencing material available, then this material can be used to fence a polygon that has a perimeter of 20 feet.

When solving geometry problems, it is very important to draw a sketch. A sketch will provide the students with a visual aid that will make solving the problem a much easier task.

The perimeter of a square can be represented by the formula $P = 4s$. Remind students that this formula can be used because a square has four sides, each of the same length.

Students should continue checking their answers. Not only should they replace the solution back into the original equation to be sure that the equation was solved correctly, but they should also look at the answer and be certain that it is logical.

Students will sometimes find the correct numerical answer for a geometry problem but fail to include the units in the final response. Remind students that five feet is a very different measurement than five liters or five miles.

An isosceles triangle has two congruent sides, called legs, and another side called the base. The angles opposite the legs are called base angles. Base angles are congruent. The other angle is the vertex angle.

Viewing the diagram of the isosceles triangle, with the length of each leg labeled as x, makes it apparent that the legs have the same measure. Therefore, some students may write the equation for finding the perimeter of this isosceles triangle as $2x + (x + 2) = 32$, instead of $x + x + (x + 2) = 32$. Either form is acceptable.

Common Error Alert

Students will often find an angle or side in an application problem but neglect to study the problem to determine if they have found the required angle or side. Defining the variable clearly, correctly labeling the drawing, and re-reading the question will all help students determine what measures to give in the answer.

1 The perimeter of a square can be represented by $P = 4s$. Therefore, $18 = 4s$ and $s = 4.5$. Because the perimeter is 18 meters, the length of one side is 4.5 meters.

2 Students should begin by drawing a scalene triangle and labeling the sides as x, $2x$, and $x + 5$. Have them read the question before they begin. "What is the length of the longest side?" The solution to the equation $x + 2x + x + 5 = 33$ is given by $x = 7$. However, this is the length of the first side. Students must find the lengths of all the sides to determine which is the longest. The length of the second side is $2x$ or 14 cm. The length of the third side is $x + 5$ or 12 cm. Therefore, the length of the longest side is 14 cm.

3 The formula for the perimeter of a rectangle is $P = 2l + 2w$, where l is the length and w is the width. However, Ferd chooses x for the width, so the length can be represented by $2x$. Sketch the rectangle and label its sides: $2x + 2(2x) = 150$. $x = 25$. The width is 25 cm. The length is equal to twice that: 2(25 cm) or 50 cm.

Additional Examples

1. In an isosceles triangle, each leg is 3 m longer than the base. The perimeter is 96 m. Find the length of the base.

x = length of the base
$x + 3$ = length of each leg

$$x + 2(x + 3) = 96$$
$$x + 2x + 6 = 96$$
$$3x + 6 = 96$$
$$3x = 90$$
$$x = 30$$

The length of the base is 30 m.

2. The length of a rectangle is 4 inches more than its width. Find the length of the rectangle if its perimeter is 36 inches.

w = width of the rectangle
$4 + w$ = length of the rectangle

$$P = 2w + 2l$$
$$36 = 2w + 2(4 + w)$$
$$36 = 2w + 8 + 2w$$
$$36 = 4w + 8$$
$$28 = 4w$$
$$7 = w$$

$4 + w = 4 + 7 = 11$

The length of the rectangle is 11 inches.

Section 2

Expand Their Horizons

In Section 2, students will solve geometry problems involving angles. Students should be familiar with most of the terms in this section but may need to review some of them.

The first problem in this section is about an isosceles triangle whose vertex angle has a measure that is three times the measure of each base angle. Base angles of an isosceles triangle are congruent. Therefore, if the base angles each have a measure of x, then the vertex angle will have a measure of $3x$. The measures of the interior angles of a triangle always have a sum of 180°. Therefore, the measures of these angles can be found using the equation $x + x + 3x = 180$.

One way to make a sketch of complementary angles is to draw a right angle and split this angle into two angles. This will help students remember that the sum of these angles is ninety degrees. For the problem in the lesson, the two angles are in the ratio five to one.

Once the smaller angle is found, the larger angle can be found by either substituting 15 for the x in $5x$ or by finding the difference between 90 and 15. Using both of these methods to find the second angle provides an efficient way for students to check their work on problems such as this one.

4 One angle can be represented by x. The other angle is then represented by $x - 10$. The sum of these angles is 180°. $x + x - 10 = 180$: $x = 95$. So, 95° is the measure of the larger angle. The measure of the smaller angle is found by subtracting 10° from 95°. The smaller angle is 85°.

Connections

Architects, draftsmen, and building contractors all use geometry on a daily basis. They apply formulas for perimeter in order to determine the correct amount of materials to purchase for their projects. They also apply their knowledge of angles to assure that buildings and structures are plumb and have the intended shape.

5 If the vertex angle is x, then the base angles are each $x - 60$. The equation used to solve this problem is $(x - 60) + (x - 60) + x = 180$: $x = 100$. This is the measure of the vertex angle. The measure of the base angles can be found by subtracting 60 from 100. The measure of each base angle is 40°.

6 Students should begin by sketching a scalene triangle. Label one angle x, the second angle $2x$, and the third angle $3x$. $x + 2x + 3x = 180$: $x = 30$. $3x = 90$. The largest angle is 90°. Point out that the original sketch was of a scalene triangle. There was no way to know that the triangle in this problem is a right triangle until the problem was solved.

Look Beyond

In geometry students will study many more real-world applications. They will apply formulas for area and perimeter to circles as well as to the polygons that were in this lesson. They will also go beyond two-dimensional geometry and learn to apply formulas to three-dimensional objects.

Additional Examples

1. In an isosceles triangle, the vertex angle is three times each base angle. Find the measure of the vertex angle.

b = measure of each base angle
$3b$ = measure of the vertex angle

$b + b + 3b = 180$
$5b = 180$
$b = 36$

$3b = 3(36) = 108$
The vertex angle is 108°.

2. The measure of an angle is four more than three times its supplement. Find the angle and its supplement.

x = measure of the first angle
$3x + 4$ = measure of the supplement

$x + 3x + 4 = 180$
$4x + 4 = 180$
$4x = 176$
$x = 44$

$3x + 4 = 136$
The measures of the angle and its supplement are 136° and 44°.