4.2 teacher notes

Objectives

- Write and solve equations of one variable to solve consumer and business problems.
- Solve consecutive integer problems.
- Solve consumer and business problems using percents.

Vocabulary

Variable (Lesson 2-1)
Equation (Lesson 3-1)
Integer (Lesson 1-1)
Commission

Prerequisites

Translating sentences into algebraic equations

Changing a percent to a decimal

Finding the percent of a number

Solving multi-step linear equations

Get Started

Have students go through papers and cut out ads with percents. Ask each student to write a problem using the information in the ad. Then have students exchange problems and solve. Once everyone has completed their problem, ask them how many used variables or equations in their answers. Explain to the class that this particular lesson will focus on: (1) identifying problems that can be solved using equations, and (2) writing and solving equations.

Section 1

Expand Their Horizons

In Section 1, students will focus on using algebra in consumer and business applications. Students may not realize how much algebra is involved in a simple shopping trip. Algebra can be used to find the amount of a purchase after the sales tax. Algebra can be used to find the cost of an item after it is marked down a given percentage. Algebra can also be used to determine the cost of cell phone plans and car rentals. Stress to students that using algebra can help them spend money wisely.

In the first example, Newt is trying to determine how many hours he must work to earn \$450. This example provides an excellent opportunity to relate algebra to the students' lives. Involve students by having them determine how long it will take to earn enough money for an item that they would like to purchase.

If students become confused when writing an equation, it may be necessary to write the problem in algebraic terms underneath the words:

hourly rate × hours Newt will work = \$450

$6 \times h = \$450$

Stress the importance of writing the answer in sentence form. This helps students be sure they have answered the question.

In this scenario, Newt must work 75 hours to earn the money. Not only do students need to check this amount in the equation, but they also need to determine if this amount makes sense in the problem. Is it feasible that Newt would work 75 hours at \$6 an hour to earn \$450?

1 Ask students, "Does it make sense that 18 custom-ordered lunch boxes would cost \$102 under the given conditions?" Remind them that verifying the reasonableness of an answer is one way to check their work.

It may be helpful to explain consecutive integers using numerical examples. Begin with 1. Ask, "What is added to one to get two? What is added to one to get three?". If you did not know the beginning number you could represent it with a variable such as x. How could you represent one more than x? $x + 1$ How could you represent two more than x? $x + 2$

The same method could be used to represent consecutive odd integers or consecutive even integers. Begin by listing the first five odd integers: 1, 3, 5, 7, 9. Then let $x = 1$ and represent each integer in terms of x.

1, 3, 5, 7, 9

x, $x + 2$, $x + 4$, $x + 6$, $x + 8$

To represent the sum of three consecutive odd integers, let the smallest integer be x. The second integer will be $x + 2$ and the largest integer will be $x + 4$. The sum is represented by the expression $x + (x + 2) + (x + 4)$.

Common Error Alert

When representing the sum of consecutive integers, students often fail to include the original integer that is represented by the variable. Have students place parentheses around each integer expression and make sure the number of groups is the same as the number of integers indicated in the problem.

2 The integers can be represented by x and $x + 2$. To find the sum of these integers, add. $x + x + 2 = 62$. $x = 30$. Check: Is $30 + 32 = 62$? Yes. Are these integers both even numbers? Yes. Are they consecutive even integers? Yes.

Additional Examples

1. At Bob's Car Bonanza a certain car rents for \$20 a day plus an additional \$0.12 a mile. If Mike's cost for one day was \$47.48, how many miles did he drive?

Define the variable.

m = the number of miles Mike drove

Write an equation.

$20 + 0.12m = 47.48$

Solve the equation.

$$\begin{aligned} 20 + 0.12m &= 47.48 \\ 0.12m &= 27.48 \\ m &= 229 \end{aligned}$$

Answer the question in a sentence.

Bob drove 229 miles.

2. The sum of three consecutive even integers is 132. Find the largest integer.

Define the variable and represent the integers.

x = the smallest integer

$(x + 2)$ = the middle integer

$(x + 4)$ = the largest integer

Write an equation.

$x + (x + 2) + (x + 4) = 132$

Solve the equation.

$$\begin{aligned} 3x + 6 &= 132 \\ 3x &= 126 \\ x &= 42 \end{aligned}$$

The first integer is 42. The largest integer is represented by the expression $x + 4$.

$x + 4 = 42 + 4 = 46$

Answer the question in a sentence.

The largest integer is 46.

Section 2

Expand Their Horizons

In Section 2, students will solve business and consumer application problems that use percentages. It may be necessary to review percentages before beginning this part of the lesson. Many students may be familiar with the "percent" statement A% of B is C, where A is the percent, B is the total, and C is the number. This statement can also be written as a proportion: $\frac{A}{100} = \frac{C}{B}$.

Many sales employees work for a commission. People who sell cars, shoes, clothing, furniture, advertising, and many other items are often paid on a commission basis.

Lizzie's commission is 20%. To illustrate how commission is calculated, the lesson has Roxie assume that Lizzie sells \$500 worth of "stuff." If Lizzie actually did sell \$500 worth, then she would earn 20% of \$500. Remind students that the word "of" usually signifies multiplication. Therefore, 20% of 500 is found by multiplying 0.20 times 500.

Many students have previously been taught to solve percentage problems by using the proportion: $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$. To find 20% of 500, write the following proportion and solve it as shown:

$$\begin{aligned} \frac{20}{100} &= \frac{x}{500} \\ 10{,}000 &= 100x \\ 100 &= x \end{aligned}$$

Many commission problems will involve a base pay plus a percent of the sales. The earnings are found using the formula: base pay + percent commission × sales = earnings. The price of the item that Lizzie sold can be found using this formula by substituting \$0 for her base pay:

$0 + 0.20p = 35.$

3 This example provides an opportunity to lead students in a discussion of income taxes and other payroll deductions. Use an actual pay stub to find the percentage of deductions taken from a check. This guided practice example can also be solved using a proportion.

$$\frac{7.5}{100} = \frac{187.50}{x}$$

$$7.5x = 18750$$

$$x = 2500$$

The worker earned $2,500 before taxes.

4 Use the formula for commission: base pay + percent commission × sales = earnings $500 + 0.15b = 770$.

It may be helpful to give the students some background information in markups and wholesale prices. Because merchants buy in large quantities, they can get their goods at wholesale price. Then, they often increase the price of these goods by a set percentage markup before selling to the public.

Connections

Consumer and business applications provide an opportunity to help students connect algebra to the real world. Help students see the value of opening a savings account that earns interest as opposed to saving money in a jar at home. Also note the different types of interest: simple and compound. There are several formulas that can be used to solve these problems. A formula for simple interest is $I = prt$, where I is interest, p is principal, r is the annual percentage rate, and t is the time of the investment in years. The formula for compound interest is more complicated: $A = p\left(1 + \frac{r}{n}\right)^{nt}$, where A is the amount in the account, p is the principal, r is the annual rate, n is the number of times interest is compounded each year, and t is the time of the investment in years.

Common Error Alert

To find the wholesale price of Newt's watch, students may incorrectly find 50% of $450. Show students that a 50% markup that results in a store price of $450 is represented by the equation $p + 0.50p = 450$. The solution $p = 300$ gives the correct wholesale price.

5 The price of the sweater + 8% of the price = 81

$p + 0.08p = 81$

Students may try to find 8% of 81 and then subtract that amount from 81. After finding the answer to this application, $75, show students that adding 8% of 75 and 75 will result in 81.

Look Beyond

In future lessons students will learn to graph linear equations. For example, the equation $500 + 0.12s = 608$ describes the relationships among the following quantities: Lizzie's monthly base pay of $500, her commission of 0.12, a month's sales represented by s, and her earnings of $608 for *a particular month*.

A more general equation can be written to describe these relationships for *any month* by using another variable in place of 608. If we use y to represent earnings for any month, the equation is $500 + 0.12s = y$. This more general equation can be graphed. (The familiar variables in graphing are x and y, so they are used now.) The graph of $500 + 0.12x = y$ is a line; 500 is the y-intercept of the line, 0.12 is the slope of the line, and for any point (x, y) on the line, x represents a month's sales and y represents that month's earnings.

Additional Examples

1. Maggie earns 12% of her sales. If she earned $180, what was the amount of her sales?

$$\frac{\text{Percent}}{100} = \frac{\text{part}}{\text{whole}}$$
$$\frac{12}{100} = \frac{180}{x}$$
$$12x = 18000$$
$$x = 1{,}500$$

The amount of Maggie's sales was $1,500.

2. Landon paid $252 for a new stereo that was on sale for 30% off. How much money did he save?

p = original price of the stereo

$$p - 0.30p = 252$$
$$0.70p = 252$$
$$p = 360$$

Now find the savings.

$360 - 252 = 108$

Landon saved $108.