4.1 teacher notes

Objective

- Write sentences as algebraic equations.

Vocabulary

Variable (Lesson 2-1)
Equation (Lesson 3-1)

Prerequisites

Translating word phrases into algebraic expressions

Get Started

Have students break into pairs. Have one student state a numeric equation such as "three plus four equals seven." The second student should write the equation using numerals and symbols. After each student has gone twice, have one student reread the equation replacing one of the numbers with the words "a number." The other student should then rewrite the equation using a variable to represent the unknown number. Explain to students that in this lesson they will focus on rewriting verbal sentences as equations using variables, numerals, and symbols.

Section 1

Expand Their Horizons

In Section 1, students will translate word sentences into algebraic equations. It is this skill that makes algebra useful in the real world. In most cases, people are not required to solve bare equations to solve everyday problems at home and work. Instead, they must assess a situation and formulate an equation to help solve the problem.

Take some time to review the key words for addition, subtraction, multiplication, and division. Remind students that these words do not form a complete list. For example, "How much greater than . . ." will usually indicate subtraction.

Now is a good time to emphasize the difference between an expression and an equation. An expression can be compared to a phrase. Just as a word phrase is not a complete thought because there is no verb, an expression is not a mathematical sentence because there is no comparison symbol. Remind students that a comparison symbol can be $=$, $<$, $>$, $\leq$, $\geq$, or $\neq$. An equation is a mathematical sentence whose comparison is $=$.

When translating a word sentence into an equation, it is usually easier for the student to first find the phrase that indicates the equal sign and then translate the phrases that belong on either side of the equal sign. An additional strategy is to use markers or highlighters to indicate the different parts of the sentence. For example, have students highlight or circle numbers in red, words or phrases for variables in green, words or phrases for operations in blue, and the equality verb in yellow. Then have them read through the sentence again and write out the symbols, variables, and numerals for each part to make the equation.

In the example, two times four plus seven equals 15, most students will be able to translate this phrase into an equation. Help students see that the next example, two times a number plus seven is equal to 15, is the same as the first, except the four has been replaced by the words, "a number." Remind students that they have already translated phrases like "two times a number plus seven." Now they simply have to insert "$= 15$" at the end of the expression they write for that phrase.

The phrases, "a number minus eight equals two" and "eight less than a number is equal to two" are represented by the same equation.

Write, "$x - 8 = 2$." Allow students time to study the equation along with the words, "eight less than a number is equal to two." If students are confused by the order of the terms, use numeric examples. Ask, "What is eight less than ten? How did you find the answer? What is eight less than 14? How did you find that answer?" Lead them to the conclusion that eight less than a number is represented by the expression "$x - 8$." Students may continue to write subtraction phrases in the wrong order. Have them replace the words "less than" with the words "subtracted from." This should help them place the numbers in the correct order.

The next example, eight decreased by a number is equal to two, may seem to be the same to the students who are not careful. Stress that it is not. Say to students, "Find eight decreased by four. Find eight decreased by six." Students should see that now they need to write the eight first, because they are subtracting **from** eight. The correct equation is $8 - x = 2$.

"Is the same as" is a phrase that represents an equal sign.

1 "Product" indicates multiplication. "Is the same as" indicates an equal sign. Point out to students that in this problem and many others, almost any letter is acceptable to be used as the variable. Mathematicians generally avoid certain letters. For example, i is not a good choice for a variable because it is often used to represent the imaginary number $\sqrt{-1}$. Roxy chose r as her variable, so her equation is $6r = 12$.

2 Have students find the "equals" and write the equation in two parts. First, write "twice a

number increased by one" as $2y + 1$. Next, write "equals five" as $= 5$. The equation is $2y + 1 = 5$.

3 Remind students to be careful about order when the phrase "less than" is used. "four less than six times a number" is written as $6x - 4$.

Students may have difficulty seeing the difference in "the sum of five times a number and two" and "five times the sum of a number and two." Point out that the first expression says "five times a number" and the second expression says "five times the sum." Note to students that phrases like "the sum of" and "the difference of" should indicate that the following numbers are grouped within parentheses with the correct operation before any other operations are shown.

To write "five times the sum," the students must first write the sum. The sum of a number and two can be written as $x + 2$. Five times that sum is $5(x + 2)$. Parentheses must be used to show that the entire sum is multiplied by five.

4 "Three times the sum" indicates that a quantity is multiplied by three. Write the sum of a number and five as $x + 5$. Use parentheses to indicate that this entire sum is multiplied by three.

5 Students may require extra time to write this equation. Students may have difficulty seeing that the first n is one factor and the sum of n and 6 is the other factor. Parentheses are needed for the quantity $n + 6$. $n(n + 6) = n^2$.

Additional Examples

1. Translate into an equation: five less than half a number is 10.

The equal sign is represented by "is." First, choose a variable. Then, translate the phrase on the left of the equal sign. Five less than half a number is written as $\frac{1}{2}n - 5$. The expression on the right of the equal sign is 10.

Half	a number	less	five	is	10.
$\frac{1}{2}$	n	$-$	5	$=$	10

2. Translate into an equation: six times the sum of a number and three is the same as 20.

The equal sign is represented by "is the same as." Choose a variable. Since the phrase is "6 times the sum," first write "the sum of a number and 3" as $y + 3$. six times this sum is $6(y + 3)$. Finally, write "is the same as 20" as $= 20$.

$6(y + 3) = 20$

six times	the sum of a number and three	is the same as	20.
6	$(y + 3)$	$=$	20

Section 2

Expand Their Horizons

In Section 2, students will be writing equations for real-world situations. Have students continue to look for key phrases to translate. Remind students that the information may be presented as more than one sentence, but there is only one translated equation. At first, some students may need to write the information as one complete sentence before translating it into an equation.

Although "making a deposit" is not one of the phrases used to recognize addition, most students will see that when a deposit is made money is added to the account. Ask, "What

phrase could be used to show that money is subtracted from a savings account?"
withdrawn, wrote a check

In the second example, there is no phrase or key word to indicate that the operation is to be multiplication. Remind students that a dozen is 12, then have them write "the cost of one egg: c" and "the cost of 12 eggs: $12c$." If twelve eggs cost \$1.09, then $12c = \$1.09$.

Ask students, "What is 3 yards more than two times its width?" It is the length of the field. This equation will have two variables. l is the length of the field and w is its width. l = 3 yards more than twice the width. $l = 2w + 3$.

In the next example, have students work one step at a time.

y = Shena's age

$y + 3$ = Deena's age

Once the variables are defined in this manner, "the sum of their ages" can be written as $y + y + 3$.

Common Error Alert

In problems such as the previous one, students often will leave out the age of the person represented by the variable only. They will write $y + 3 = 27$ instead of writing $y + y + 3 = 27$. Having them define each person's age will help eliminate this error.

6 Remind students that the value of a dime is \$0.10, therefore the number of dimes (d) multiplied by the value of one dime is the total amount, \$7.50.

7 Note that the first sentence helps you assign the variable and write expressions for both people's ages. The second sentence tells you how to write the equation that can be solved to answer the problem.

Additional Examples

1. Write an equation to represent the following situation: Sandy paid twice as much for her hamburger as she paid for her french fries. Her total order was \$3.45.

Define the variable.
f = amount paid for french fries
Represent the other quantity in terms of the variable.
$2f$ = amount paid for hamburger
"Her total order" indicates that these quantities are added.
$f + 2f = \$3.45$

2. Write an equation to represent the following situation: The base of a triangle is three more than its height. Five times the sum of the base and the height is 30.

Define the variable.
h = height of the triangle
Represent the other quantity in terms of the variable.
$h + 3$ = base of the triangle
The sum of the base and the height is $h + h + 3$.
$5(h + h + 3) = 30$