3.6 teacher notes

Objectives

- Use formulas to find the value of one of the variables in the formula.
- Solve formulas for a given variable in the formula.
- Use formulas to solve real-world applications.

Vocabulary

Formula
Variables (Lesson 2-1)
Equation (Lesson 3-2)
Algebraic expression (Lesson 2-1)
Value

Prerequisites

Simplifying expressions with rational numbers

Solving one-step linear equations

Solving two-step linear equations

Solving multi-step linear equations

Evaluating expressions

Get Started

Allow students to brainstorm and suggest formulas that will help them see the relevance of this lesson. One way to facilitate this process is to provide an activity. Divide the class into two groups. Give the groups four minutes to come up with as many formulas as they can. Then, compare their answers. Using the distance formula, show them how to think through rewriting a formula.

- The distance a person travels is given by the formula $d = rt$, where d = distance, r = rate, and t = time.
- Set up an equation to find out how fast a person is traveling if the person goes 60 miles in 2 hours. $60 = 2r$
- How would you solve the equation? Divide 60 by 2
- Using this information, if you know the distance and the time, how can you find the rate? $r = \frac{d}{t}$

Section 1

Expand Their Horizons

In Section 1, students will rewrite formulas so that they can more easily solve for a given variable.

The addition of the Greek Letter π may confuse some of the students. Lead the students in a discussion relating constants and variables. Although π is actually a constant, it maintains its symbolic form when rearranging formulas rather than being converted to a numerical value. When these formulas are solved for numerical values, π may be kept to provide an exact answer. Sometimes 3.14 or $\frac{22}{7}$ is used to give a numeric approximation.

Before the lesson, it may be helpful to review the concepts of inverse operations and the order of operations. Remind students that they will be working the problems using the order of operations backwards just as they did when solving equations. The difference here is that the final solution will be a variable equal to an algebraic expression rather than a numerical value.

Students will probably be very familiar with geometric formulas. Use the formula for the area of a triangle to demonstrate why rearranging formulas can be useful. Write the two formulas $A = \frac{1}{2}bh$ and $2\frac{A}{b} = h$ on the board. Ask students: "If you knew the base and the height, which formula would be most useful to find the area? Why?" Responses should lean towards the first formula because it is easier to substitute the numbers for the variables and simplify. Then ask: "If you knew the area and length of the base, which formula would be used to find the height? Why?" The answers should now lean toward the second formula because it is easier to use.

In the first example, the goal is to find the width of the rectangle. By following along with the video, students should see that the width was found by dividing the area, 12, by the length, 6. They should then be able to generalize this knowledge to predict that the width of a rectangle can be found by dividing its area by its length. Use this knowledge to help students follow the example when $A = lw$ is solved for w.

1 Use the formula $w = \frac{A}{l}$. $\frac{24}{8} = 3$. The width is 3 inches. Be sure that students include the appropriate units when using formulas.

Common Error Alert

When rewriting formulas, students will often solve for the wrong variable. Tell students to treat the variable that they are asked to solve for as the only variable. Students who are having difficulty can circle the variable they are attempting to isolate or temporarily replace it with a symbol like a star or happy face to make it stand out more.

Sometimes it is helpful for students to first work through problems that have values for all variables except the one for which they are solving. For instance, when trying to solve the formula $P = 2l + 2w$ for w, first let l be 2 and let p be 10. Have the students find the width as they would normally do and note each step. Then, have the students rewrite the formula in terms of w and note how similar operations were performed to solve both problems.

To rewrite the formula $P = 2l + 2w$ and solve for w, use the backwards order of operations. Do the subtraction first. $P - 2l = 2w$. Next, do the division. $\frac{P - 2l}{2} = w$.

2 Use the formula $\frac{P - 2l}{2} = w$. When finding the value of a variable, be sure students use the order of operations in forward order.

Additional Examples

1. Solve $P = 4s$ for s.

$P = 4s$
$\frac{P}{4} = \frac{4s}{4}$
$\frac{P}{4} = s$

2. Solve $E = mc^2$ for m.

$E = mc^2$
$\frac{E}{c^2} = \frac{mc^2}{c^2}$
$\frac{E}{c^2} = m$

Section 2

Expand Their Horizons

In Section 2, students will rewrite formulas with rational numbers. Students will use the formula for changing Fahrenheit degrees to Celsius degrees and the formula for changing Celsius degrees to Fahrenheit degrees extensively in their science classes. The formula for changing Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. The freezing point of water is 0°C or 32°F. The boiling point of water is 100°C or 212°F. The conversion formulas are based on these benchmark temperatures. Students can substitute them into the formula to verify it.

Rewriting the Fahrenheit to Celsius formula to solve for Celsius is a two-step process. The order of operations must be applied in reverse. Remind students to group the expression $F - 32$ before continuing. Next, do the division. Remind students that division of fractions is done by multiplying the reciprocal. Therefore, to divide by $\frac{9}{5}$, it is necessary to multiply by $\frac{5}{9}$. The equation becomes $\frac{5}{9}(F - 32) = C$. This equation can also be written as $\frac{5(F - 32)}{9}$.

3 Solve $d = rt$ for r. This problem was briefly addressed in the Getting Started section. To solve for r, divide both sides of the equation by t. $\frac{d}{t} = \frac{rt}{t}$; $\frac{d}{t} = r$.

4 Notice that the $\frac{1}{3}$, the π, and r^2 are all multiplied by the h. To undo this multiplication, divide. However, dividing by $\frac{1}{3}$ is the same as multiplying by 3. Do this step first. $3V = \pi r^2 h$. Next, it is acceptable to divide by the π and the r^2 at the same time. $\frac{3V}{\pi r^2} = h$.

Look Beyond

Mastery of the skill of rewriting formulas will enable the students to proceed to topics such as solving applications, graphing linear equations, and solving systems of equations with confidence. Rewriting formulas is particularly useful when working with linear equations. For instance, it is often useful to use the slope-intercept form of a linear equation when graphing lines. This formula is $y = mx + b$. However, many equations are not given in this form and may need to be rewritten with the y variable isolated.

Additional Examples

1. Solve $Ax + By = C$ for x.

Use inverse operations to isolate x in the formula.

$Ax + By = C$

$Ax = C - By$

$\frac{Ax}{A} = \frac{C - By}{A}$

$x = \frac{C - By}{A}$ or $x = \frac{C}{A} - \frac{By}{A}$

2. Solve $s = \frac{1}{2}(a + b + c)$ for c.

Eliminate the fraction and then isolate c using inverse operations.

$s = \frac{1}{2}(a + b + c)$

$2s = a + b + c$

$2s - a = b + c$

$2s - a - b = c$ or $c = 2s - a - b$