3.5 teacher notes

Objectives

- Solve equations involving more than one step.
- Solve multi-step equations involving fractions.
- Solve multi-step equations using the Distributive Property.
- Solve equations that are identities.
- Solve equations that have no solution.

Vocabulary

Multi-step equation
Variable (Lesson 2-1)
Like terms (Lesson 2-4)
Identity
Distributive Property (Lesson 2-3)
Least common multiple (Lesson 1-3)

Prerequisites

Simplifying expressions with rational numbers

Solving two-step linear equations

Using the Distributive Property to simplify expressions

Get Started

- Have students play a memory game together. Give every student a list of the vocabulary words for this lesson and any others you wish to add (up to 10 vocabulary terms) and that many index cards.
- Have the students write an example of a term on one side of a card for all the terms.
- Then, have pairs of students shuffle their cards together and place them face down in an array on a flat surface.
- Students take turns turning over pairs of cards trying to find the two examples of a vocabulary term. For each vocabulary match, they get to remove it from the table and receive a point.

Section 1

Expand Their Horizons

In Section 1, students will solve multi-step linear equations. Solving multi-step linear equations involves using more than one inverse operation to solve more complex equations. Encourage them to see that each step for solving a multi-step linear equation will be an application of a skill that they have already learned.

A Manipulatives Section is provided for this lesson. Students will expand their skills with modeling and solving linear equations of one variable with algebra tiles. Beginning with the Manipulatives Section will give students a concrete foundation for the algebraic processes used to solve multi-step linear equations. Students who have difficulty with the abstract skills may be able to use algebra tiles on some, but not all, of the Independent and Additional Practice problems.

Remind students of the balance idea. For the first example, $2x + 3x = 10$, use a variable weight that is twice as heavy as a constant weight. Show 5 variable weights set in a group of 2 and a group of 3 in one pan and 10 constant weights in the other pan. Students should see that they need to combine the $2x$ and the $3x$ rather than subtract one of them to the other side of the equation.

In the equations whose solutions are all real numbers, the students can see that in the end both pans have the same quantities in each pan. Therefore, they still balance. In the equations whose solutions are "no solution", the students will see that in the end the pans are unbalanced and that there is no way to make them balanced.

There are several ways in which students can check their work. One of the most common methods is to substitute the solution into the original equation and simplify. If the resulting equation is true, the solution is correct.

Solving the equation $6p + 5 = 8p + 1$ with manipulatives may increase students' understanding. Begin with 6 "x" tiles and 5 "one" tiles on the left side of the mat. Have eight "x" tiles and 1 "one" tile on the right side of the mat. Remove 6 "x" tiles from both sides of the mat. This leaves $5 = 2p + 1$. Next, remove 1 "one" tile from each side of the mat. This leaves $4 = 2p$. Divide into two equal parts. The solution is $p = 2$.

Common Error Alert

To solve equations such as $9x + 5 - x = 4x + 3$, students will often begin by adding x to both sides of the equation instead of combining like terms. Setting the problem up on either a balance or with algebra tiles will help students see that the x is subtracted from the $9x$, leaving $8x + 5 = 4x + 3$.

The equation $y + y + 1 + y + 2 = 3y + 3$ simplifies to be $3y + 3 = 3y + 3$. When both sides of the equation are exactly the same, the equation is an identity. The equation is true for all real numbers.

The next equation simplifies to become $-3 = 7$. There is no solution to this equation. This can be written as "∅".

1 There are variables on both sides of the equation. An alternative method to solve this equation so that the variable is on the right is:

$$\begin{aligned} 4B + 2 &= 37 - B \\ 2 &= 37 - 5B \\ -35 &= -5B \\ 7 &= B \end{aligned}$$

2 This equation simplifies to the identity $7 = 7$. The equation is true for all values of the variable, so the solution is all real numbers.

Additional Examples

1. Solve: $5y - 8 = 2y + 1$

Subtract $2y$ from both sides and add 8 to both sides. Find y by dividing both sides by 3.

$$5y - 8 = 2y + 1$$
$$3y - 8 = 1$$
$$3y = 9$$
$$y = 3$$

2. Solve: $x - 3x + 2 + 4x = 2x + 7 - 3 + 5$

Group like terms on both sides. In this case, the equality becomes untrue when $2x$ is subtracted from both sides.

$$x - 3x + 2 + 4x = 2x + 7 - 3 + 5$$
$$2x + 2 = 2x + 9$$
$$2 = 9$$

There is no solution.

Section 2

Expand Their Horizons

In Section 2, students will solve multi-step equations using the Distributive Property and solve multi-step equations with rational numbers.

Using a balance or algebra tiles, the first equation will have four sets of $(3m - 2)$ and one tile on the left side of the balance. This gives students a better understanding of the Distributive Property.

It may be necessary to review multiplication of fractions and whole numbers. During the example $\frac{1}{2}j - 6 = -20 - \frac{2}{3}j$, show the operations in detail.

Review the term "least common multiple." Remind students that they are finding the least common denominator of the terms. They also may need to be reminded that the denominator of a whole number is one.

3 First use the Distributive Property; $13z = 48 - 3z$. Add the $3z$ to both sides of the equation; $16z = 48$. $z = 3$.

4 Use the Distributive Property on both sides of the equation. When simplified, the equation becomes $12 = 2$. It has no solution. Some students may see that the equation has no solution at the step $4t + 12 = 4t + 2$. The coefficients of the variables are the same, but the constants are different.

5 Although it is not necessary, the easiest way to solve equations involving fractional coefficients is to eliminate them by multiplying each term by the least common multiple. In this case the least common multiple is 18. $\frac{1}{6}w = 2 - \frac{1}{9}w$

$$18\left(\tfrac{1}{6}w\right) = 18\left(2 - \tfrac{1}{9}w\right)$$
$$3w = 36 - 2w$$
$$5w = 36$$
$$w = \frac{36}{5}$$

Common Error Alert

Students will often multiply the fractions by the least common multiple, but neglect to multiply the whole numbers, or they will multiply one side of the equation by the least common multiple but not the other. Stress the Multiplication Property of Equality. If the equation is going to remain balanced, all terms must be multiplied by the same number.

Additional Examples

1. Solve: $2(4t - 3) = -3(2t + 1)$

Use the Distributive Property and then solve.

$$\begin{aligned} 2(4t - 3) &= -3(2t + 1) \\ 8t - 6 &= -6t - 3 \\ 14t &= 3 \\ t &= \frac{3}{14} \end{aligned}$$

2. Solve: $\frac{2}{3}x + 4 = 3x - \frac{1}{2}$

Multiply all terms by 6, which is the least common multiple of the two denominators, and then solve.

$$\begin{aligned} \frac{2}{3}x + 4 &= 3x - \frac{1}{2} \\ 6\left(\frac{2}{3}x + 4\right) &= 6\left(3x - \frac{1}{2}\right) \\ 4x + 24 &= 18x - 3 \\ -14x &= -27 \\ x &= \frac{27}{14} \end{aligned}$$