3.4 teacher notes

Objectives

- Solve two-step equations.
- Check solutions.
- Determine if a number is a solution for a two-step equation.
- Provide reasons for each step in solving a two-step equation.

Vocabulary

Equation (Lesson 3-2)
Inverse (Lesson 3-1)
Addition Property of Equality (Lesson 2-3)
Subtraction Property of Equality (Lesson 2-3)
Multiplication Property of Equality (Lesson 2-3)
Division Property of Equality (Lesson 2-3)

Prerequisites

Simplifying expressions with rational numbers

Solving one-step linear equations

Identifying and apply properties of equality

Supplying reasons for a simple algebraic proof

Get Started

Have one student describe a simple task, such as tying a shoe or wrapping a present. Have the student write step-by-step what should be done. Have another volunteer describe how to undo the task. In most cases, the student will start at the **bottom** of the list to undo whatever procedure has been described. After the example is finished, show the class how the same procedure applies to algebra. When solving equations, work backwards to undo operations in the original equation.

Section 1

Expand Their Horizons

In Section 1, students will solve two-step linear equations by working backwards. In earlier grades students may have used "working backwards" as a problem solving strategy. Clarify that in this lesson the concept of "working backwards" refers to using the reverse of the order of operations.

Remind students that they have isolated variables with one operation before. When they solved a problem like $x + 3 = 23$, students subtracted eight from both sides of the equation to isolate x and get the solution $x = 20$. The same initial step is done for $5x + 3 = 23$, but the resulting equation is $5x = 20$. In this case the students have isolated the variable term. They must divide by five to isolate the variable and solve for x; $x = 4$. Now is a good time for students to get in the habit of always checking solutions and making sure that they are reasonable.

A Manipulatives Section is offered for this lesson. It focuses on combining the skills learned in previous lessons to use algebra tiles to model and solve two-step linear equations. By having students solve some equations visually and concretely, it may assist them in transferring those skills to the abstract procedures for problems that cannot be solved concretely.

Common Error Alert

Some students may try to solve equations using the order of operations. It might be a good idea to solve one of the equations by doing the multiplication or division first. Allow the students to see that although it is possible, it is much more inconvenient. For instance, the equation $4x - 5 = 11$, could be solved by first dividing by 4, to get $x - \frac{5}{4} = \frac{11}{4}$. Then $x = \frac{16}{4}$, or $x = 4$. Doing a problem in this manner should allow them to see the benefits of working backwards by using the reverse of the order of operations.

To solve the equation $5x + 3 = 23$, begin with 5 "x" tiles and three "one" tiles on the left side of the mat. There should be 23 "one" tiles on the right side of the mat. Take three one tiles from each side of the mat. $5x + 3 - 3 = 23 - 3$; $5x = 20$. Align the "x" tiles vertically and divide the remaining "one" tiles into 5 equal rows. Write, "$\frac{5x}{5} = \frac{20}{5}$." Take away all but one row; $x = 4$.

To solve $\frac{M}{6} - 10 = -12$, begin at the end of the order of operations. Add –12 and 10. This is –2. Next, multiply by 6. $-2(6) = -12$. This can be done in a horizontal fashion. $M = (-12 + 10) \cdot 6$. $M = -12$. Working a few examples like this will help students understand the steps that are involved in solving these equations.

Some two-step problems are in the form $\frac{x - 3}{2} = 5$. These problems have a tendency to confuse students. They want to do the addition first. However, even if the parentheses are not present the quantity in the numerator is grouped. If the equation is solved using the backwards order of operation, multiplication is done before parentheses. This problem is solved by multiplying five and two, and then adding the three. The solution is $x = 13$.

1 To find out if a number is a solution to an equation, check the equation by substituting the number for the variable.
$-7(2) - 10 = -14 - 10 = -24$. $-24 \neq -4$

2 Explain how to solve the equation $\frac{p}{5} + 9 = 13$. Use the order of operations backwards. $(13 - 9) \cdot 5 = p$

3 Solve $-4J - 1 = 11$. Have students solve this equation using the proper steps. It is still important for students to check their solutions.
Some students may prefer this method to show addition or subtraction:

$$\begin{aligned} -4J - 1 &= 11 \\ +1 \quad & \quad +1 \\ \hline -4J &= 12 \\ \frac{-4J}{-4} &= \frac{12}{-4} \\ J &= -3 \end{aligned}$$

Check:
$$\begin{aligned} -4J - 1 &\stackrel{?}{=} 11 \\ -4(-3) - 1 &\stackrel{?}{=} 11 \\ 12 - 1 &\stackrel{?}{=} 11 \\ 11 &= 11 \checkmark \end{aligned}$$

Additional Examples

1. Is $b = 2$ a solution to the equation: $4b - 5 = 3$?

Substitute the value of two for the variable, b.

Check: $4b - 5 = 3$

$4(2) - 5 = 3$

$8 - 5 = 3$

$3 = 3$ ✓

Two is a solution to $4b - 5 = 3$.

2. Solve $\frac{r}{5} + 7 = -3$

Use two steps to find the value of r. First subtract seven from both sides and then multiply both sides by five.

$$\frac{r}{5} + 7 = -3$$

$$\frac{r}{5} + 7 - 7 = -3 - 7$$

$$\frac{r}{5} = -10$$

$$\frac{r}{5} \cdot 5 = -10 \cdot 5$$

$$r = -50$$

Section 2

Expand Their Horizons

In Section 2, students will explain the reasons for steps used to solve two-step linear equations when some of the details are not given. These problems are basically like the algebraic proofs students did in Lesson 2-3. However, students should now have the ability to supply both the steps and the reasons. They should have a much greater understanding of the concepts involved.

In the lesson example, students see that $5x + 3 = 23$ is the Given statement. In the next statement, $5x = 20$, the three was missing from the left side and the right side was 20. Three was subtracted from both sides. The Subtraction Property of Equality was the reason for this statement. Remind students that subtraction undoes addition. In the last statement, $x = 4$, five was missing from the left side and 20 was replaced by four. This reason was the Division Property of Equality. Division undoes multiplication.

4 Remind students to focus on the operation being preformed to both sides of the equation. If students use the operations shown in either expression, they will identify the incorrect property.

Connections

Phone companies and electric companies sometimes use two-step equations or formulas to calculate what customers owe. A cell phone company will charge a set rate plus a given amount for each minute used above the base amount. An electric company will charge a set amount plus a set rate for each kilowatt-hour of electricity used. For instance, an electric company may charge a set rate of $8.00 plus $0.18 a kilowatt hour. If a person uses 1,000 kilowatt hours of electricity, the charge would be $8 + 0.18(1{,}000) = c$; $c = \$188$. On the other hand, if a person gets a bill for $229.40, he or she can check the bill with the formula, $8 + 0.18(k) = 229.40$. In this case $k = 1{,}230$. If the bill is for 1,230 kwh, then it has been computed correctly.

Additional Examples

1. Explain the steps used to solve the equation $\frac{b}{-4} - 2 = 3$.

$\frac{b}{-4} - 2 = 3$ Given

$\frac{b}{-4} = 5$ Addition Property of Equality

$b = -20$ Multiplication Property of Equality

2. Explain the steps used to solve the equation $3y + 7 = 28$.

$3y + 7 = 28$ Given

$3y = 21$ Subtraction Property of Equality

$y = 7$ Division Property of Equality