3.3 teacher notes

Objectives

- Solve one-step equations using addition and subtraction.
- Solve one-step equations using multiplication and division.
- Check one-step equations using addition, subtraction, multiplication, and division.

Vocabulary

Equation (Lesson 3-2)
Solution (Lesson 3-2)
Coefficient (Lesson 2-1)

Prerequisites

Simplifying expressions with rational numbers

Translating word phrases into algebraic expressions

Identifying and apply properties of equality

Solving equations by inspection

Get Started

Write the following equations on the board:

$2m = 12$ $\qquad\qquad$ $\frac{b}{3} = 5$

Ask students to guess what value of the variable will make each equation true. $m = 6$ and $b = 15$

Ask the students to examine the equations again. Ask a volunteer to identify what operation they could use to "get rid" of the operation associated with the variable for each equation. division, multiplication.

Now write:

$\frac{2m}{2} = \frac{12}{2}$ $\qquad\qquad$ $\frac{b}{3} \cdot 3 = 5 \cdot 3$

$m = 2$ $\qquad\qquad$ $b = 15$

Explain that the operations of multiplication and division can "undo" each other to isolate a variable.

Section 1

Expand Their Horizons

In Section 1, students will solve one-step equations using multiplication and division. Most students will be able to solve the equations presented in this lesson by inspection. Some students may resist efforts to write steps to solve the equations. To encourage students to learn to write steps, show them a problem such as this:

$4x - (2x + 5)(3) = 2(x - 3) + 7$

Ask students if they know the solution by inspection. They should not. Tell students that in this lesson they will learn the process of how to solve problems like these. First, they will learn the process on simpler equations, and then they will use the same process on more complex equations one step at a time.

Using the processes that are presented in this lesson, the solution to the above equation is:

$$\begin{aligned} 4x - (2x + 5)(3) &= 2(x - 3) + 7 \\ 4x - 6x - 15 &= 2x - 6 + 7 \\ -2x - 15 &= 2x + 1 \\ -4x &= 16 \\ x &= -4 \end{aligned}$$

Stress the inverse operations being used to isolate the variable for each step of the solution.

Students may not fully understand fractions. Show students that multiplying by $\frac{1}{3}$ and dividing by 3 are the same. $15 \cdot \frac{1}{3} = 5$; $15 \div 3 = 5$. Help students see that the fraction $\frac{x}{5}$ is the same as $\frac{1}{5}x$ by rewriting the expression as $\frac{1x}{5}$.

If students are still struggling with the idea of undoing operations, it might be a good idea to solve problems such as these or to refer to the Getting Started section to refresh their memories.

1. $2 + -2$
2. $-6 + 6$
3. $\frac{1}{2} \cdot 2$
4. $\frac{3}{4} \cdot \frac{4}{3}$

Using a balance to represent equality, select a "five" weight to represent the variable and a "one" weight to represent the constant. If possible, use a different color for either positive or negative. To represent the equation $x + 2 = 7$, place an x weight and two constant weights on one side of the balance and place seven constant weights on the other side of the balance. Write the equation on the board. Show how subtracting two from both sides of the equation can be represented by removing two constant weights from each side of the balance. Then, there will be only a variable weight in one pan and the constant weight in the other pan. The solution is $x = 5$. Choose another equation depending on the weights available. This method helps students understand why identical operations must be performed on both sides of the equation.

Use the Manipulatives Section that is provided with the lesson to introduce the students to solving equations using algebra tiles. After they have mastered the concrete procedures, lead them through the rest of the lesson and the more abstract methods of solving equations.

When using the manipulatives, have students write each step that is represented. For instance, in the manipulative problem $3x = 12$, begin by having 3 "x" tiles on the left side of the mat and 12 "one" tiles on the right side of the mat. On paper, write, "$3x = 12$." Next, when the "x" tiles are aligned vertically, and the "one" tiles are aligned horizontally in three rows, write $\frac{3x}{3} = \frac{12}{3}$. When all tiles except one row containing an "x" tile on the left side of the mat and four "one" tiles on the right side of the mat have been removed, write "$x = 4$." The equation looks like this:

$$\begin{aligned} 3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

Most examples in the video do not lend themselves to using manipulatives because there are either decimals in the equation or the numbers are too large. If students are dependent upon the manipulatives, help them visualize the setup for the problems in the video or draw representations on paper.

1 $5x = 65$. The operation that undoes multiplication is division. $\frac{5x}{5} = \frac{65}{5}$. $x = 13$. It is important that students continue to check solutions.

The next example is looking for the number that can be divided by 8 to get 15.

Common Error Alert

In problems such as $\frac{g}{8} = 15$, students may divide 15 by 8 to try to find the solution. The number, g, that is divided by 8 to get 15 is the same number as 15 times 8.

2 $\frac{z}{4} = 3$. The operation that undoes division is multiplication. $z = 4 \cdot 3$. $z = 12$.

Additional Examples

1. Solve and check the equation:
$7x = 84$

Use the Division Property of Equality to divide both sides by 7.

$$7x = 84$$
$$\frac{7x}{7} = \frac{84}{7}$$
$$x = 12$$

Check: $7x \stackrel{?}{=} 84$
$7(12) \stackrel{?}{=} 84$
$84 = 84$ ✓

2. Solve and check the equation:
$-2x = 18$

Use the Division Property of Equality to divide both sides by –2.

$$-2x = 18$$
$$\frac{-2x}{2} = \frac{18}{-2}$$
$$x = -9$$

Check: $-2x \stackrel{?}{=} 18$
$-2(-9) \stackrel{?}{=} 18$
$18 = 18$ ✓

Section 2

Expand Their Horizons

In Section 2, students will solve one-step equations using addition and subtraction.

Remind students that $m - 20$ is the same as $m + (-20)$. This may help students see that $m - 20 + 20 = m$.

3 A good problem to solve using manipulatives is $y - 5 = 12$. Begin with the "x" tile and five red "one" tiles on the left side of the mat. Have 12 yellow "one" tiles on the right side of the mat. This is represented by the equation $y - 5 = 12$. Create five "zero pairs" on the left side of the mat by adding five yellow "one tiles" to each side of the mat. Stress that in order to keep the mat balanced, any operation that is performed on one side, must also be performed on the other side. This is represented by $y - 5 + 5 = 12 + 5$. Remove the zero pairs. Group all tiles on the right side of the mat. The equation becomes $y = 17$. Check: $17 - 5 = 12$.

In the example to find baking time, Newt tried to write the equation as $P + 75 = 2$ because the entire time was 2 hours. Remind students to use consistent units in all calculations for any given problem. In this case, since the baking time was 75 minutes, the total time was written as 120 minutes.

4 The equation, $t + 7 = 9$, can be easily solved using manipulatives. Begin by representing the problem with one "x" tile and 7 "one" tiles on the left side of the mat and 9 "one" tiles on the right side of the mat. Remove 7 "one" tiles from each side of the mat, representing the equation, $t + 7 - 7 = 9 = 7$. The solution is $t = 2$.

Look Beyond

The study of mathematics revolves around solving equations. Students will solve linear equations with many steps, quadratic equations, exponential equations, trigonometric equations, radical equations, integral equations and many others. The processes for solving these equations will all be based upon a firm foundation in solving one-step linear equations.

Connections

Although most people do not think of it as solving equations, it is something they do daily. Suppose a group of four teenagers went out for pizza. At the end of the meal, they get a check totaling $21.35. They decide to split the bill evenly and pay $25.00 including the tip. They need to know what times four will give them $25.00, they will use the inverse of multiplication to divide $25 by four to get the amount each will have to pay for the meal. Each person will have to pay $6.25.

Additional Examples

1. Solve and check the equation: $\boldsymbol{n - 3 = 7}$

Isolate n by adding 3 to both sides.

$$n - 3 = 7$$
$$n - 3 + 3 = 7 + 3$$
$$n = 10$$

Check: $n - 3 \stackrel{?}{=} 7$

$$10 - 3 = 7$$
$$7 = 7 \checkmark$$

2. Solve and check the equation: $\boldsymbol{r + 7 = 8}$

Isolate r by subtracting 7 from both sides.

$$r + 7 = 8$$
$$r + 7 - 7 = 8 - 7$$
$$r = 1$$

Check: $r + 7 \stackrel{?}{=} 8$

$$1 + 7 = 8$$
$$8 = 8 \checkmark$$