3.1 teacher notes

Objectives

- Recognize and use the Reflexive, Symmetric, and Transitive Properties of Equality.
- Recognize and use the Addition, Subtraction, Multiplication, and Division Properties of Equality.
- Supply the reasons for an algebraic proof when solving a simple equation.

Vocabulary

Reflexive Property of Equality
Symmetric Property of Equality
Transitive Property of Equality
Addition Property of Equality
Subtraction Property of Equality
Multiplication Property of Equality
Division Property of Equality
Equation
Algebraic proof

Prerequisites

Simplifying expressions with rational numbers

Identifying algebraic properties

Translating word phrases into algebraic expressions

Get Started

- Begin with the equation written on the board $5 = 5$.
- What number makes this equation true? $5 + 2 = 5 +$ ___ 2
- What number makes this equation true? $5 - 3 = 5 -$ ___ 3
- Which operation makes this equation true? $5 \cdot 7 = 5$ ___ 7 Multiplication
- Which operation makes this equation true? $5 \div 4 = 5$ ___ 4 Division
- After the lesson is complete, ask students to go back to each question and identify the property of equality.

Section 1

Expand Their Horizons

In Section 1, students will increase their understanding of equality and learn properties of equality. The properties of equality are very important when completing algebraic proofs and solving equations. The concept of an equation being a balance will help students understand equations. Use a balance scale to demonstrate many of the properties.

The Reflexive Property of Equality can be related to the reflection of a figure on a graph. Both objects are the same shape and have the same area. Just as an image and its reflection are congruent, the two identical expressions in a reflexive equation have the same value. Show this by placing two identical objects on either pan of a balance scale.

The Symmetric Property of Equality will be used extensively by students. When an equation is in the form $9 = 2x + 1$, most students will find the equation easier to solve if they rewrite it as

$2x + 1 = 9$. To demonstrate this, place two different quantities with different types of weights but the same total weight. Let each quantity represent a unique expression. Show students that regardless of how the quantities are placed in the pans, there is always a balance.

Common Error Alert

Students often confuse the Reflexive and Symmetric Properties of Equality because of their similarity. The Reflexive Property of Equality shows the same expression on either side of an equation. The Symmetric Property of Equality shows different expressions on either side of the equal sign.

1 The variable on the right side of the equation is identical to the variable on the left side of the equation. This is the Reflexive Property of Equality.

Students may enjoy extending the "age example" in the video to the classroom. Use three students in the class who are the same age and replace Newt's, Roxie's, and Lizzie's names with the names of students in the class.

Students may relate well to the Transitive Property of Equality by using pattern blocks as concrete models and compare color, shape, or number of sides. In other words, for any three green shapes say, "If Block 1 is the same color as Block 2, and Block 2 is the same color as Block 3, then Block 1 is the same color as Block 3." This is also a strong argument in logic.

2 The Reflexive Property can be used to write $9 = 9$. Write the three equations together in a string: $x = 9 = 9 = y$. Remove the extra 9 and equal sign to make a statement about x and y. Therefore, $x = y$.

It will be beneficial to spend time studying the formal definitions of these properties. This will allow students an opportunity to understand and use algebraic language. Students who have difficulty distinguishing the properties may want to make flash cards for each property with the name on one side and the formal algebraic definition and example on the other side.

In an equation, the equal sign works like a fulcrum on a balance. An equation is always balanced. In order to keep an equation balanced, any operation that is performed on one side of the equation must also be performed on the other side of the equation.

The properties of equality can be represented by a balance that has the same number of objects in each pan. The students will notice that it is in the balanced position. Show the device becoming off balance when objects are changed in only one pan. It is helpful to write equations representing the objects in the balance to demonstrate mathematically what is taking place when the number of objects in the balance is changed.

- The Addition Property of Equality can be represented by adding the same quantity to each pan.

- The Subtraction Property of Equality can be represented by taking the same quantity from each pan.
- The Multiplication Property of Equality can be represented by doubling or tripling the quantity in each pan.
- The Division Property of Equality can be represented by halving or thirding the quantity in each pan.

3 It is important that students not leave out the statement, $c \neq 0$. Students may need to be reminded that division by 0 is not possible in the real number system.

Additional Examples

1. State the property of equality illustrated in the following: If $c = 2$, then $2 = c$.

Ask students if the two expressions in either equation are the same. They are not, so the Reflexive Property is not shown. Then note that there are only two expressions and their position in the equation has been changed. This must be the Symmetric Property of Equality.

2. State the property of equality illustrated in the following: If $x = 5$, then $x - 2 = 5 - 2$.

Ask students to identify the operation used in the second equation, how was each expression changed. Both expressions had two subtracted, so the Subtraction Property of Equality is demonstrated. Note to students that they can simplify the right hand side of the equation $(5 - 2)$ and still have a true statement. So if $x = 5$, then $x - 2 = 3$.

Section 2

Expand Their Horizons

In Section 2, students will be using the properties of equality to supply reasons for algebraic proofs. When supplying reasons for the algebraic proofs, it may be necessary for some students to actually write the process used, like subtracting four, or multiplying by three. This will enable them to recognize the property that has been used.

To find the correct property of equality, the student must determine which operation has been performed. Take this time to remind students about inverse operations. In these problems, students can see the steps used to solve equations and have a feel for all the steps without having to solve equations themselves.

Common Error Alert

Make sure students focus on what has been done from one equation to the next, not what is happening in a single expression. For example, moving from the equation $x + 4 = 10$ to $x = 6$ some students will identify the Addition Property of Equality because x and 4 were added in the first expression. However, the step to get from the first equation to the second is subtraction. So the Subtraction Property of Equality was used.

Explain to students that properties of equality are applied in every step of solving equations. At any step, students can state which property they used. Students will learn to solve equations on their own but may not be required to identify the properties they used; but, it is good practice to know what the properties are and how they are used. There are other properties of equality not taught in this lesson, which students will be introduced to when appropriate.

The equation is always given at the beginning. Therefore, the reason for the original equation is, "Given."

In the step $2x = 14$, it may be easier for students to see that 10 has been added to 4, than to see that 10 has been added to $2x - 10$. Help them understand this process. Ask them to simplify the expression $2x - 10 + 10$. They should recognize the use of inverse operations and that the result is only $2x$. If students are still struggling, have them use the rules of addition and subtraction for integers and rewrite the expression as $2x + (-10) + 10$. At this point most students will see that adding the opposites –10 and 10 results in 0. These students may need to review Lesson 1-2.

To undo the operation $2 \cdot x$, use the inverse operation, division. $2 \div 2 = 1$; therefore, $2x$ divided by 2 is $1x$. If the left side of the equation is divided by two, in order to maintain balance, the right side of the equation is also divided by 2. The number 14 divided by 2 is 7. The reason for the final step, then, is the Division Property of Equality.

4 When $3x - 5 = 1$ is changed into $3x = 6$, the "minus 5" disappears from the left side of the equation. How is a five used to change a one into a six? It is added: $1 + 5 = 6$. Also, $-5 + 5 = 0$. This equation illustrates the Addition Property of Equality.

Look Beyond

All of the branches of mathematics have underlying properties, axioms, tautologies, postulates, or identities. Mathematicians use all of these premises to prove new and interesting theorems and create explanations to better understand both academic and applied mathematics. Proofs are used in all branches of mathematics including geometry, trigonometry, calculus, topology, and many more. The properties of equality in algebra are only the beginning. There are properties that deal with sets of numbers and operations. Students have been exposed to some of these, including the Associative, Commutative, and Distributive Properties.

Additional Examples

1. Consider the following:

Steps	Reasons
$2x + 3 = 9$	________
$2x = 6$	________
$x = 3$	________

What is the reason for each step?

Step 1: Given, this is the original equation
Step 2: Think: How did $2x + 3$ become $2x$? by subtracting 3. Check the expression on the other side of the equation: $9 - 3 = 6$. The reason is the Subtraction Property of Equality.
Step 3: Think: How did $2x$ become x? by dividing by 2. Check the expression on the other side of the equation: $\frac{6}{2} = 3$. The reason is the Division Property of Equality.

2. Consider the following:

Steps	Reasons
$\frac{m}{2} - 4 = 3$	________
$\frac{m}{2} = 7$	________
$m = 14$	________

What is the reason for each step?

Step 1: Given
Step 2: Think: How did $\frac{m}{2} - 4$ become $\frac{m}{2}$? by adding 4. Check the expression on the other side of the equation: $3 + 4 = 7$. The reason is the Addition Property of Equality.
Step 3: Think: How did $\frac{m}{2}$ become m? by multiplying by 2. Check the expression on the other side of the equation: $7 \times 2 = 14$. The reason is the Multiplication Property of Equality.