20.2 teacher notes

Objectives

- Find experimental probability.
- Find theoretical probability.
- Find the probability of the complement of an event.

Vocabulary

Event
Probability of an event
Experimental probability
Theoretical probability
Trial
Certain event
Impossible event
Outcome
Law of Large Numbers
Complementary events
Mutually exclusive events

Prerequisites

Reducing fractions to lowest terms

Adding and subtracting fractions

Get Started

- Make a two-column table on the board. Label the left column “Red” and the right column “Black.” Select one student who will be the recorder.
- Place three red checkers and seven black checkers into a bag. (Do not let the students see how many of each are in the bag.) Have a student randomly select one checker from the bag and announce the color of the selected checker. Tell the recorder to put a tally mark under the appropriate column and the student to put the checker back and to pass the bag to the next student. Repeat the process until ten students have selected a checker. Students should shake the contents of the bag between draws.
- Ask the class what fraction of the selected checkers was red. Record this fraction and tell them that they have just found an **experimental probability**.
- Show the class that there are three red checkers and seven black checkers. Ask if anyone knows the **theoretical probability** of drawing a red checker at random. If a student says $\frac{3}{10}$, verify that he is correct. If no student says $\frac{3}{10}$, tell the class that the theoretical probability is $\frac{3}{10}$.
- Tell the class that they will learn more about experimental and theoretical probability in today’s lesson.

Section 1

Expand Their Horizons

In Section 1, students will first be introduced to the basic concepts of probability, and then they will learn the difference between experimental and theoretical probability.

Probability is described as the likelihood of an event. Students will most likely already have ideas about probability because it is widely used throughout the real world. For example, weather forecasters often announce the probability of rain or snow. In this lesson, probabilities are written as fractions in lowest terms. However, probabilities in the real world are often given as percents. You may wish to remind students that a fraction can be converted to a percent by dividing the numerator by the denominator and multiplying by one hundred.

The probability of an event can range from zero to one inclusive, meaning that the probability of an event can be any number from zero to one, *including* zero and one. If the probability of an event is zero, then there is absolutely no way for that event to occur. This is called an "impossible event". An example of an event with a probability of zero could be selecting a black marble from a bag that only contains blue and red marbles. If the probability of an event is one, then the event will definitely occur. This is called a "certain event". An example of an event with a probability of one could be selecting a red marble from a bag that only contains red marbles. Most events in the real world have a probability somewhere between zero and one, or between 0% and 100% if percents are used. An event becomes more likely to happen as its probabilities get closer to one or 100%. When the probability of rain is 90%, more people take an umbrella when they go outdoors than when the probability of rain is 10%.

Experimental probability is defined as the number of successful trials divided by the total number of trials in an experiment. If the experiment consists of flipping a coin 10 times, then each flip is a trial. It is important to realize that the experimental probability of an event is only true for that particular experiment. When the same experiment is repeated, the experimental probability may change. Suppose an experiment consisting of 10 coin-flips yields three heads. Then the experimental probability of flipping a coin and getting a head is $\frac{3}{10}$, which can be written as $P(\text{head}) = \frac{3}{10}$. However, a second experiment of flipping a coin 10 times may yield four heads, so for that experiment, the probability of flipping a coin and getting a head is $\frac{4}{10}$, or $\frac{2}{5}$.

Because experimental probability can change when the same experiment is repeated, some students may say that it is useless, but in fact, experimental probability is very useful. For instance, a business owner may use experimental probability, using the number of times something occurred in the past to predict the number of times it will occur in the future. If in one year, 65% of a business's customers are females under the age of 30, then the business owner may expect about the same percent of next year's customers to be female and under 30 and will tailor the advertising as such.

Theoretical probability, on the other hand, is defined as the number of favorable outcomes divided by the total number of outcomes. Outcomes are possible results of an experiment. In the experiment of flipping a fair coin, there are two outcomes because there are two possible results: heads or tails. The event of flipping a coin and getting a head has one favorable outcome, heads, and two total outcomes, so the theoretical probability of flipping a coin and getting a head is $\frac{1}{2}$. In *theory*, one half of a very large number of flips will result in heads. However, for just a few flips of the coin, the number of heads may not be half of the number of flips.

The Law of Large Numbers states that as the number of trials in an experiment increases, the experimental probability gets closer to the theoretical probability. In the lesson, the DVD characters first conduct six trials and find the probability of selecting a blue marble to be $\frac{1}{2}$. Their next experiment consists of 300 trials, and the experimental probability increases to $\frac{2}{3}$, which happens to be the same as the theoretical probability. It is unlikely though, that in reality, they would get the exact theoretical probability after just 300 trials.

Historical Note: During the Second World War, the South African mathematician John Kerrich spent his time in a prison camp by flipping a coin 10,000 times. Of those flips, 5,067 were heads. His experimental probability of flipping a coin and getting a head, 50.67%, was very close, but not equal to, the theoretical probability of 50%. In the early 1900's, the English statistician, Karl Pearson, flipped a coin 24,000 times, and 12,012 were heads. His experimental probability of 50.05% was even closer to the theoretical probability.

Use the following table to answer Questions 1 and 2. A fair die was rolled 20 times. The number of times each number landed face up is shown.

Number	Number of times face up
1	4
2	2
3	3
4	5
5	2
6	4

1 **Find the experimental probability of rolling a four.** The number of successful trials is the number of times a four was rolled. Four was rolled five times. The total number of trials is the number of times the die was rolled. The die was rolled 20 times. The experimental probability of rolling a four, therefore, is $\frac{5}{20}$, which reduces to $\frac{1}{4}$.

2 **Find the theoretical probability of rolling a four.** The number of favorable outcomes is the number of faces that show a four. Only one face has four on it. The total number of outcomes is the number of possible results in the experiment. There are six possible results; the die has six faces, numbered one through six. The theoretical probability of rolling a four, therefore, is $\frac{1}{6}$. Notice that the experimental and theoretical probabilities are different. If more trials were conducted, the experimental probability of rolling a four would likely be closer to $\frac{1}{6}$. If students doubt this, it may be worthwhile to conduct more trials of rolling a die. The last five minutes of each class period, for a few weeks, can be spent on this. Keep a running total of the number of times a four was rolled, as well as the total number of rolls. At the end of each session, adjust the experimental probability as needed. (It may be helpful to write the probability as a decimal for this experiment.) Students will get a better sense of the Law of Large Numbers when they see the probability get closer to $\frac{1}{6}$, or $0.1\overline{6}$, with each passing day.

Additional Examples

1. **A spinner with five equal sectors labeled one through five was spun 35 times. Use the results on the right to find the experimental probability of spinning a two. Express the answer as a fraction and as a percent rounded to the nearest tenth of a percent.**

Number	Times
1	7
2	8
3	9
4	5
5	6

The number of successful trials is the number of times a two was spun. Divide that by the total number of trials, which is the total number of spins.
$P(2) = \frac{8}{35} \approx 22.9\%$

2. **Use the information in Question 1 to find the theoretical probability of spinning an even number. Express the answer as a fraction and as a percent.**

A favorable outcome is spinning an even number. Divide the number of sectors labeled with two or four by the total number of sectors.
$P(\text{even}) = \frac{2}{5} = 40\%$

Section 2

Expand Their Horizons

In Section 2, students will find the probability of the complement of an event. Complementary events are defined as mutually exclusive events; one of which must happen.

Mutually exclusive events are events that cannot happen at the same time. In a standard deck of cards, the events of drawing a queen and drawing a king are mutually exclusive. A card cannot be a queen and a king at the same time. However, the events of drawing a queen and drawing a diamond are not mutually exclusive. A card can be a queen and a diamond at the same time.

So, an example of complementary events is drawing a queen and drawing a card that is not a queen. They are clearly mutually exclusive, and if a card is drawn, one of the two events must happen. The card is either a queen or it is not. Because one of the two events must happen, the sum of their probabilities is one or 100%. Therefore, if the probability of one of the two events is known, the probability of the other event is easily found by subtracting the first probability from one or 100%.

In more advanced probability problems, finding the probability of the complement of an event is often easier than finding the probability of the event itself. Once the complement of an event is found, the probability of the event itself is found by simply subtracting the probability of the complement from one or 100%. Note that some books refer to the complement of A as A'.

3 **The probability of winning a carnival game is $\frac{3}{25}$. Find the probability of NOT winning the game.** The events of winning a game and not winning a game are complements. Because one of the two must occur, the probability of not winning the game is one minus the probability of winning.

$$P(\text{not winning}) = 1 - \frac{3}{25} = \frac{25}{25} - \frac{3}{25} = \frac{22}{25}$$

The probability of not winning is $\frac{22}{25}$ or 88%.

Winning a game and losing the game are not always the only possible outcomes. Some games, such as soccer, can end in a tie. Therefore, it is important to be careful not to use the word "losing" as the complement of "winning." However, it is always correct to say that "winning" and "not winning" are complements.

Connections

Probability is used throughout the scientific community. Examples include finding the probability a random egg hatches and reaches maturity, finding the probability of an earthquake in a given area, and finding the probability that a rabbit will be all white.

Look Beyond

The concept of probability is one way to describe the likelihood of an event. Another way is the concept of odds. Because students may study odds in future math courses, a solid understanding of theoretical probability now will enable them to easily grasp the concept of odds later. Probability and odds are different ways to give the same information. For example, suppose a bag contains three red checkers and seven black checkers. The probability of drawing a black checker at random is $\frac{7}{10}$, while the odds in favor of drawing a black checker are 7:3.

Additional Examples

1. **Suppose the probability of a certain child having brown eyes is $\frac{1}{4}$. What is the probability that the child does not have brown eyes?**

   The events of having brown eyes and not having brown eyes are mutually exclusive. Because one of the two must occur, the events are complementary.

   $P(\text{not brown}) = 1 - P(\text{brown})$

   $P(\text{not brown}) = 1 - \frac{1}{4}$

   $P(\text{not brown}) = \frac{3}{4}$

2. **In a bubble gum machine, the gumballs are either blue, yellow, red, or orange. The probability of getting a blue gumball is 15%. Find the probability of getting a gumball that is yellow, red, or orange.**

   Getting a gumball that is yellow, red, or orange can be thought of as getting a gumball that is not blue.

   $P(\text{not blue}) = 100\% - P(\text{blue})$

   $P(\text{not blue}) = 100\% - 15\%$

   $P(\text{not blue}) = 85\%$

   $P(\text{yellow, red, or orange}) = 85\%$