2.5 teacher notes

Objectives

- Evaluate algebraic expressions for given numerical values.
- Use the order of operations.

Vocabulary

Algebraic expression (Lesson 2-1)
Variable (Lesson 2-1)
Square root (Lesson 1-4)
Absolute value (Lesson 1-2)

Prerequisites

Simplifying expressions with rational numbers

Simplifying expressions with exponents, roots, and absolute values

Applying the order of operations

Get Started

- Draw a square. Label one side of the square 3 inches.
- Ask, "Can anyone tell me how to find the perimeter of the square?" Find the product 4(3).
- Ask, "What is the perimeter of the square?" 12 inches
- Draw a square. Label one side of the square *s*.
- Ask, "What is an expression for the perimeter of the square?" 4*s*
- Ask, "What would we need to know to find an actual perimeter of the square?" A numerical value for *s*.
- Tell the class, "Today we are going to evaluate algebraic expressions like 4*s*, using numerical values for the variables."

Section 1

Expand Their Horizons

In this lesson, students will be evaluating algebraic expressions. Assure students that evaluating expressions is something they have already seen. Students are probably familiar with distance/rate/time applications. If the students know a rate of speed and a time, they can evaluate the expression rt to find a distance traveled. For example, if $r = 45$ miles per hour and $t = 2$ hours, then distance $= rt = 45 \cdot 2 = 90$ miles.

Explain that numbers are being substituted for variables when evaluating expressions. It may be helpful to use the same color chalk for both the variable and its value to visually show the connection in the substitution. For example, if students are evaluating the expression $x + 1$ for $x = 3$, write, "$x = 3$", and the x in the expression in blue chalk.

The Total Bases example at the beginning of the lesson provides students with a real-world application which is represented by an algebraic expression and then that expression is evaluated.

When evaluating $(-2)^n$, students must be especially careful to insert the parentheses, especially if using a calculator. $(-2)^n$ and -2^n are equal for odd values of n, but not necessarily equal for even values of n.

For $n = 3$, $(-2)^n = (-2)^3 = (-2)(-2)(-2) = -8$, and $-2^n = -2^3 = -(2)(2)(2) = -8$.

For $n = 4$, $(-2)^n = (-2)^4 = (-2)(-2)(-2)(-2) = 16$, and $-2^n = -2^4 = -(2)(2)(2)(2) = -16$.

Emphasize that although $(-2)^n$ and -2^n are equal for odd values of n, they represent different multiplication expressions.

When evaluating expressions it is important that students remember to use the order of operations. A review of these skills may be necessary. Before beginning the lesson, it may be helpful to review exponents, roots, and absolute value.

In the expression, $\sqrt{3^2 + 4^2}$, the square root symbol acts as a grouping symbol, so the powers are evaluated first. $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

1 Ask, "What number used as a factor three times is equal to -8?" $(-2)(-2)(-2) = -8$. The answer is -2.

2 $|3 - (-8)| = |3 + 8| = |11| = 11$.

Connections

People in all walks of life evaluate expressions every day. Consumers must evaluate expressions to find the sales tax on a purchase or to find the sale price of an item that is a given percent off the original price.

Car owners evaluate expressions to find the average gas mileage for a vehicle.

Gas mileage = Number of miles driven/Gallons of gasoline used.

Consumers also must be able to evaluate area formulas in order to determine how much paint or wall paper to buy when remodeling a home or office.

Most students are interested in issues that involve money, but some may not be familiar with compound interest. Compound interest is the interest paid on both the original principal and on any previously earned interest. Explain what the expression in the lesson represents. The expression $P\left(1 + \frac{r}{n}\right)^{nt}$ is used to compute an amount in an account that earns compound interest, where P is principal, or initial amount, r is the annual interest rate, n is the number of times per year the interest is compounded, and t is the number of years.

Evaluating this expression gives students the opportunity to practice applying the order of operations. If $10,000 is deposited in an account that earns interest at an annual rate of 6%, compounded monthly, then the amount in that account after 5 years would be: $P\left(1+\frac{r}{n}\right)^{nt} = \$10{,}000\left(1+\frac{0.06}{12}\right)^{12\cdot 5} = \$10{,}000(1+0.005)^{60} = \$10{,}000(1.005)^{60} = \$13{,}488.50$. Note that the factors 12 and 5 must be multiplied to get the exponent 60.

It may be helpful to contrast compound interest and simple interest. An expression that can be used when an account earns simple interest is $P(1 + rt)$. For the same deposit, but using simple interest instead of compound interest, the amount in the account would be: $P(1 + rt) = \$10{,}000(1 + 0.06 \cdot 5) = \$10{,}000(1.30) = \$13{,}000.00$

Look Beyond

Although the history of algebra began in ancient Egypt and Babylon where people learned to solve linear and quadratic equations, ancient mathematicians used only occasional abbreviations in their algebraic expressions. By medieval times Islamic mathematicians talked about arbitrarily high powers of the unknown *x*, and worked out the basic algebra of polynomials, but still did not use modern symbols. It was not until the 16th century that mathematicians began using symbols for the unknown and for algebraic powers and operations.

Additional Examples

1. Evaluate $\frac{x^3y^2}{xy^4}$ for $x = -3$ and $y = 2$.

$$\frac{x^3y^2}{xy^4} = \frac{(-3)^3(2)^2}{-3(2)^4} = \frac{-27(4)}{-3(16)} = \frac{-108}{-48} = \frac{9}{4}$$

2. Evaluate $-|x^3 - 3|$ for $x = -4$.

$$-|x^3 - 3| = -|(-4)^3 - 3| = -|-64 - 3| = -|-67| = -67$$